# Application of Dynamic Systems Family for Synthesis of Fuzzy Control with Account of Non-Linearities

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***Abstract.*** *Dynamic system with nonlinearities has been considered. This system has been divided into a set of linear subsystems. A fuzzy controller of the considered system has been synthesized. It takes into account nonlinearities of the system and provides smooth switching between controllers of the linear subsystems. An unstable subsystem has been utilized, which provides better dynamic characteristics of the considered system. Comparison with traditional controller has been conducted. Corresponding qualitative and quantitative estimates have been provided. They testify the expediency of the proposed approach.*

## Keywords

***Dynamic system, fuzzy controller, nonlinearity, unstable subsystem.***

## 1. Introduction

The vast majority of systems that exist in a nature are nonlinear. In the design of controllers of such systems, they use, in particular, the approaches of classic linear control theory. Namely, the existing nonlinearities in the system are, at one point, rejected or linearized, and the control inputs are designed for the resulting linear system. Then these control inputs are used in the studied nonlinear system. Unfortunately, this approach may not always ensure the desired transients in the system.Moreover, the controllers designed this way generally do not consider the design constraints on the values of the system's intermediate coordinates.

Today there are many up-to-date methods of design of nonlinear system controllers. Among these approaches, one can distinguish, in particular, feedback linearization, geometric methods, LMI-based methods, etc. The methods applying the fuzzy sets apparatus have become widely used (see for instance [1]). In particular, in the case of discrete nonlinear systems with constant coefficients, some interesting results are obtained in [2]. In [3] it is suggested to use polynomial fuzzy model based system for approximation of the nonlinear system. It has been proved that this approach allows maintaining the system's stability. However, the possibility of improving the dynamic characteristics of nonlinear systems with application of fuzzy controller has not been investigated in the paper mentioned above. Observer-based approach was used for robust control synthesis of nonlinear system in [4]. In [5] Takagi-Sugeno fuzzy controller has been used for synthesis of dual-mode fuzzy predictive control scheme for discrete system. In the case of induction motor, inverse fuzzy model based torque control has been studied in [6]. But the linearization technique, which is proposed there, does not fully take into account peculiarities of the nonlinear components that are present in the investigated system.

This article first describes the general approach to the design of nonlinear system controllers. These theoretical manipulation scan be applied to the design of linear systems controllers. In this case, the object model is the same in all subsystems of the family. In case of nonlinear systems, we have suggested an approach to the synthesis of controllers that ensure the desired behavior of the system. The paper considers the case when a controller synthesized for one of the subsystems ensures its unstable behavior.

## 2. Description of Systems Using the Family of Dynamic Systems

Let us consider a nonlinear system, which is generally described by the $n^{th}$ order differential equation. It can be reduced to the system of the first order differential equations

$$\bar{x}'(t) = f(\bar{x}(t), \theta(t)) + g(\bar{x}(t))\bar{u}(t) + \xi(t), \qquad (1)$$

where $\bar{x}(t) = [x_1(t), x_2(t), \ldots x_n(t)]^T$, $x_1(t) = x(t)$, $x_2(t) = x'(t)$, $\bar{u}(t) \in R^n$ - is the vector of control inputs, $\bar{\xi}(t)$ - are the external disturbance inputs, $f(\bar{x}(t), \theta(t))$ and $g(\bar{x}(t))$ - are the nonlinear functions described in the field of the system's operating points, $\theta(t)$ - is the vector of variable parameters that do not depend on $x(t)$. This system, ignoring external disturbances and using the first term of Taylor expansion by of the right-hand side, can be linearized in several points. As a result, we get a linear model with variable parameters. The working domain has been divided into subdomains according to the number of linearization points. Therefore, the model of the $i$-th system will look like Eq. (2).

$$\dot{\bar{x}}(t) = \mathbf{A}_i + \mathbf{B}_i u(t). \qquad (2)$$

Thus, in fact, we received a family of linear dynamic systems Eq. (2).

The general nonlinear system model has been created with a set of fuzzy rules of the form:

$$R^i: \text{IF } x_1 \in \mathbf{M}_1^i \ i \ x_2 \in \mathbf{M}_2^i \ i \ \ldots \ x_n \in \mathbf{M}_n^i \text{ THEN } \dot{\bar{x}}(t) = \mathbf{A}_i\bar{x}(t) + \mathbf{B}_i\bar{u}(t), \quad i = \overline{1,\mathrm{n}}, \qquad (3)$$

where $R^i$ – $i$-th rule, $\mathbf{M}_j^i, \mathbf{N}_j^i$, $i = \overline{1,k}$, $j = \overline{1,n}$ - regions of partition, $\mathbf{A}_i, \mathbf{B}_i \in R^{x \times x}$ - matrices that form a system's model around some operating point (local model).

The case of feedback control for each of the subsystems has been studied in the paper. The controller of the general system has been obtained by using fuzzy logic apparatus.

$$R^i: \text{IF } x_1 \in \mathbf{N}_1^i \ i \ x_2 \in \mathbf{N}_2^i \ i \ \ldots \ x_n \in \mathbf{N}_n^i \text{ THEN } \bar{u}(t) = \mathbf{K}_i\bar{x}(t), \quad j = \overline{1,\mathrm{n}}, \qquad (4)$$

where $\mathbf{K}_i \in R^{x \times x}$ - matrices determining the settings of the $i$-th system controller to a particular standard linear form.

The general system's model can be obtained using, for instance, the gravity defuzzification method.

The theorem has been proved to investigate the stability of such systems, see [7].

**Theorem 1.** *If we select matrices* $\mathbf{K_i}, i = \overline{1,k}$ *so that the systems Eq. (2) are Lyapunov stable and*

$$\sum_{j=1}^{k-1}\sum_{i=j+1}^{k}\left\{\left[(\mathbf{K}_i - \mathbf{K}_j)(\nu_j\mu_i\mathbf{B}_j - \nu_i\mu_j\mathbf{B}_i)\right]^T \times \right. \\ \left. \times\, \mathbf{P} + \mathbf{P}(\mathbf{K}_j - \mathbf{K}_i)(\nu_j\mu_i\mathbf{B}_j - \nu_i\mu_j\mathbf{B}_i)\right\} \\ > -\sum_{i=1}^{k}\bar{\mathbf{Q}}_i, \qquad (5)$$

where $\bar{Q}_i > 0$, $i = \overline{1\ldots k}$, $\mathbf{P}$ matrix is connected with all the systems, $\mathbf{P} = \prod_{i=1}^{k}\mathbf{P}_i$, the system Eq. (1) is also Lyapunov stable.

The following paragraphs consider the issues of the controllers synthesis for the obtained subsystems.

## 3. Synthesis of the System Controller with the Account of Nonlinearities

Most actual systems are nonlinear. Therefore, the accuracy of the model, and consequently, the accuracy of the synthesized controller increases with the increasing number of nonlinearities, which are simultaneously considered in the system. In the electrotechnical systems for approximation of nonlinearities, the function is traditionally approximated with a line $f(u) \approx f_1(u) = \mathbf{K}_{tp}^1 u$, by expansion this nonlinear function in a Taylor series at the origin and ignoring the add ends above the second-order. However, this approximation is valid only in a small region of input voltage variation.

To approximate the function, it has been proposed in [8] and [9] to apply the fuzzy logic apparatus. Depending on the input signal value, nonlinearity is approximated as follows:

$$f(u) \approx \sum_{i=1}^{n}\mu_i(u)\mathbf{K}_{tp}^i u, \ i = \overline{1..n}, \ \mathbf{K}_{tp}^i > \mathbf{K}_{tp}^{i+1}, \qquad (6)$$

where $\mu(u)$ - is the membership function that is defined below. Application of the fuzzy logic apparatus ensures smooth switching between the functions that approximate the nonlinear function at different points.

Takagi-Sugeno fuzzy controller was applied to implement the proposed approach [10]. For the linguistic variable "argument" and its terms $\mathbf{U}_i$, $i = \overline{1\ldots\mathrm{n}}$ it has been suggested in [11] to apply a triangular membership function. Defuzzification will be conducted with a center of gravity method. Then:

$$u(\bar{t}) = \sum_{i=1}^{n}\mu_i(t)\, g_i(\bar{x}). \qquad (7)$$

The rule base has the following form:

$$\text{IF } (u \text{ in } \mathbf{U}_i) \text{ THEN } \bar{u}(t) = g_i(\bar{x}), \qquad (8)$$

where $g_i(\bar{x})$ are corresponding functions ofthe state vector of the system, $\mathbf{U}_i$ - are fields of division. The corresponding controller was synthesized for each of the functions $g_i(\bar{x})$ by the full state vector, which is as follows:

$$g_i(\bar{x}) = \mathbf{K}_i \bar{x}(t), \quad i = \overline{1..n}, \tag{9}$$

where $\mathbf{K}_i$ defines the tunings for a particular standard linear form. With this approach used, only one variable is fuzzified, so the number of rules is equal to the number of divided regions. The increasing number of rules results in the increased accuracy on the one hand, while making the controller more complex on the other, as more rules need to be designed. On the contrary, there is an option of using fewer rules to simplify the controller, for instance, only for boundary and several interior points. The system's dynamics will get some what worse at that.

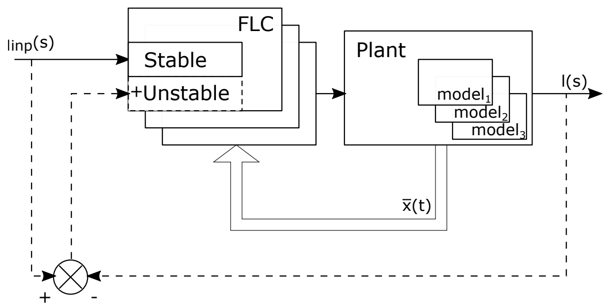



**Fig. 1:** Block diagram of the studied system.

Figure 1 shows the general scheme of the controller of the studied system. FLC component is responsible for fuzzification of the input signal in nonlinear object and contains the appropriate rules base.

## 4. Unstable Subsystem

It is possible to use control that makes subsystem unstable. To improve characteristics of the system, authors propose to form the control output of the linear system in such a way that one of its root be situated in the right hand side, i.e. subsystem is unstable. This approach is used in the domain of big deviations. The corresponding block that will form such a signal is dotted.

In addition, the studies conducted in papers [12], [13], [14] and [15] demonstrated the efficiency of unstable subsystem use, which allows for a significant improvement of the overall performance. The state variables remain within the acceptable limits at this.

In other domains, the controller that provides tuning to standard binomial form is used.

Another option is to use the controller that is tuned for a standard linear Bessel form [16] in the domain of big deviation. On one hand, the subsystem is stable but on the other, the transient improvement is smaller.

The system controller with the proposed subsystem has the following form

$$u(t) = \xi_1(t)\, u_{first}(t) + \xi_2(t)\, u_{fuz}(t), \tag{10}$$

where $u_{first}(t)$ - is the controller, which corresponds to the subsystem with greater speed, $\xi_i(t)$ - corresponding triangular membership functions.

## 5. Application of the Proposed Approach

Many technological systems use two-mass model for description, where the first mass is responsible for the integral inertia of the engine, and the second mass - for the load inertia. Detailed description of the two-mass systems is given in [17] and [18]. The model is quite well-known, it is used, in particular, in [19] and [20].

General view of the structural scheme of the studied two-mass system a long with the electric component of the engine is shown in Fig. 2.

The mathematical model of the system's electric component is described the following way

$$\begin{aligned}
&v_a(t) = R_a i_a(t) + L_a\left(\frac{di_a(t)}{dt}\right) + e_a(t),\\
&e_a(t) = K\omega_m(t),\\
&T_m(t) = K i_a(t),\\
&v_a(t) = f(v_m(t)),
\end{aligned} \tag{11}$$

where $v_a(t)\, i_a(t)$ (A) is a motor armature voltage, $R_a(t)$ ($\Omega$) - is an armature resistance, $L_a$ (H) - is an armature inductance, $i_a(t)$ (A) - is armature current, $e_a(t)$ (V) - is a back electromotive force, $K_m$ (Vc·rad$^{-1}$) - is a motor constant, $T_m(t)$ (Nm) - is a torque caused by motor, $\omega_m(t)$ (rad·s$^{-1}$) - is an angular motor rotational velocity, $f(v_m)$ - is a nonlinear function, which corresponds to the voltage thyristor converter model, and $v_m$ - is a thyristor converter input voltage.

We also disregard the nonlinearities in the mathematical model of mechanical portion, such as backlash in the gear. The linear model of the system's mechanical portion is described as follows:

$$\begin{aligned}
&J_m\left(\frac{d\omega_m(t)}{dt}\right) = (N)^2 T_m(t) - \frac{1}{N}T_s(t) - \frac{1}{N}B_m\omega_m(t),\\
&J_L\left(\frac{d\omega_L(t)}{dt}\right) = T_s(t) - B_L\omega_L(t) - T_d,\\
&T_s(t) = k_s\left(\frac{1}{N}\theta_m(t) - \theta_L(t)\right) + B_s\left(\frac{1}{N}\omega_m(t) - \omega_L(t)\right),\\
&\frac{d\theta_L(t)}{dt} = \omega_L(t),\\
&\frac{d\theta_m(t)}{dt} = \frac{1}{N}\omega_m(t),\\
&\frac{d\theta_L(t)}{dt} = \omega_L(t),\\
&l_{out}(t) = r\theta_L(t),
\end{aligned} \tag{12}$$

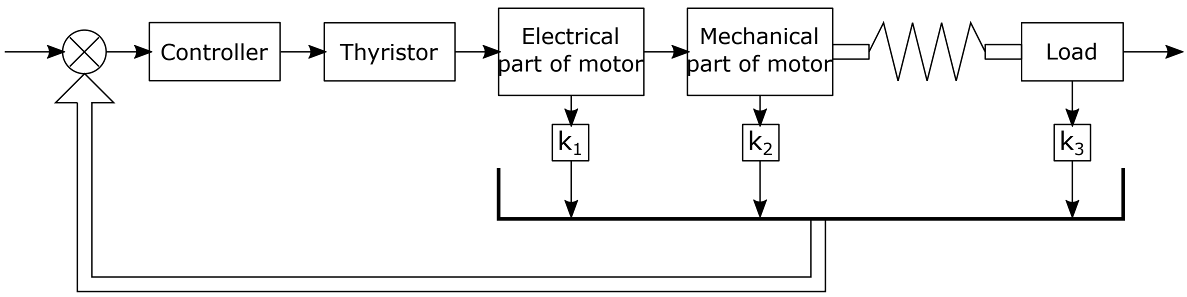



**Fig. 2:** Structural scheme of the two-mass system with the engine's electric component.

where $k_S$ (Nm·rad$^{-1}$) is an elastic stiffness, $B_m$ and $B_L$ (Nm·(rad·s)$^{-1}$) are the coefficient of internal viscous motor friction and load respectively, $B_S$ (Nm·(rad·s)$^{-1}$) is internal damping coefficient of the shaft, $J_m$ and $J_L$ (kg·m$^2$) are the moments of inertia of the motor and the working gear (load), $\omega_m$ and $\omega_L$ (rad·s$^{-1}$) are the angular rotational velocities of the motor and the end-effect or respectively, $T_m$ and $T_d$ (Nm) are the torques of the motor and the load (end-effector), $T_s$ (Nm) is the passed torque of the shaft, $1/N$ is the reduction ratio.

Actual system's model contains many nonlinear components. Consideration of all of them leads to significant complications of the simulation process. Therefore, we disregard a part of nonlinearities in this paper, in particular, Coulomb frictions, see [9]. We also disregard nonlinearities in the mathematical model of mechanical portion, such as backlash in gears. To investigate stability we use results from [7].

In current research the parameters of the investigated model where taken from the electrode motion control system of electric arc furnace [19].

The methodology that helps to obtain the corresponding coefficient is a Taylor series expansion of the nonlinearities present in the system. By means of linearization of model Eq. (11), we can get linear models with different coefficients. In the last equation from Eq. (11) nonlinear function can be presented as, $f(v_m(t)) = k_i v_m$ where $k_i$ are constant coefficients, the value of which depends on the point of linearization (thyristor converter input voltage in this case). Using several points of expansion, we get a set of such coefficients. By means of fuzzy logic we can provide a smooth transition from one coefficient to another depending on the state of the system. I.e. using Eq. (7), Eq. (9) we can utilize membership functions presented in Fig. 3.

Based on the model of the system and approaches described in Sections 2. and Section 3. , we get different forms of fuzzy controller. Coefficients from

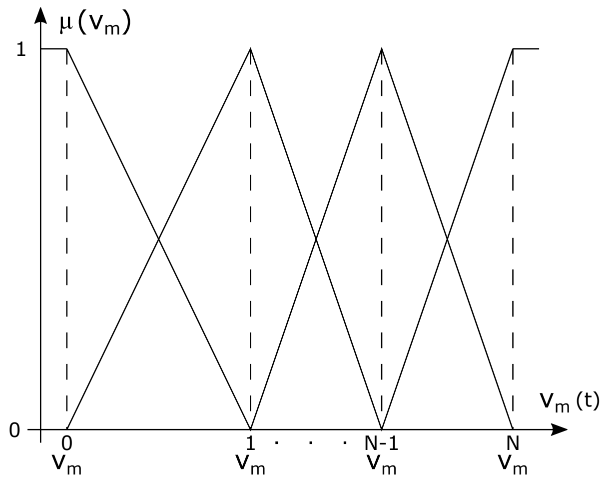



**Fig. 3:** Triangle membership function.

Eq. (9) can be obtained as in [20], [21], [22], [23] and [24]. These coefficients for the case of controller tuned to standard binomial form are given below. Coefficients for other forms used in this research were also obtained. This means that using the coefficients Eq. (12) we can obtain smooth control signal, which will take into account both shape and value of the corresponding signal. When using unstable subsystem we can take into account an error signal and use a triangle membership function to obtain necessary transient gain. This is done in the similar way.

In the Eq. (17) $\mathbf{K}_p^i$ - are coefficients that are obtained after Tailor serial expansion of the initial nonlinear model and are different for each $i$-th linear model; they correspond to regions with centers at $v_m^i$.

In the case of backlash, using the proposed controller in Section 4. we can improve the transients and obtain smooth acceleration.

For this research, a Matlab model of the investigated system has been used.

$$k_1 = -\frac{\left(-5J_L J_m L_a N^2\omega + B_L J_m L_a N^2 + B_m J_L L_a N^2 + B_s J_m L_a N^2 + J_L J_m R_a N^2 + B_s J_L L_a\right)}{(J_L J_m \mathbf{K}_p^i N^2)}, \tag{13}$$

$$k_2 = \frac{\begin{pmatrix} 10J_L^2 J_m^2 L_a N^4\omega^2 - 5B_L J_L J_m^2 L_a N^4\omega - 5B_m J_L^2 J_m L_a N^4\omega - 5B_s J_L J_m^2 L_a N^4\omega + B_L^2 J_m^2 L_a N^4 + \\ +B_L B_m J_L J_m L_a N^4 + 2B_L B_S J_m^2 L_a N^4 + B_m^2 J_L^2 L_a N^4 + B_m B_s J_L J_m L_a N^4 + B_s^2 J_m^2 L_a N^4 - \\ -J_L^2 J_m K_m^2 N^4 - J_L J_m^2 L_a k_s N^4 - 5B_s J_L^2 J_m L_a N^2\omega + B_L B_s J_L J_m L_a N^2 + 2B_m B_s J_L^2 L_a N^2 + \\ +2B_s^2 J_L J_m L_a N^2 - J_L^2 J_m L_a k_s N^2 + B_S^2 J_L^2 L_a \end{pmatrix}}{(J_L^2 J_m K_m \mathbf{K}_p^i N^4)}, \tag{14}$$

$$k_3 = L_a \frac{\begin{pmatrix} -B_s^2 J_L^3 J_m^2 N^4\omega^5 + 5B_s J_L^3 J_m^2 k_s N^4\omega^4 - 10J_L^3 J_m^2 k_s^2 N^4\omega^3 + 10B_L J_L^2 J_m^2 k_s^2 N^4\omega^2 - \\ -5B_L^2 J_L J_m^2 k_s^2 N^4\omega - 5B_L B_s J_L J_m^2 k_s^2 N^4\omega + 5J_L^2 J_m^2 k_s^3 N^4\omega + B_L^3 J_m^2 k_s^2 N^4 + \\ +2B_L^2 B_s J_m^2 k_s^2 N^4 + B_L B_s^2 J_m^2 k_s^2 N^4 - 2B_L J_L J_m^2 k_s^3 N^4 - B_s J_L J_m^2 k_S^3 N^4 - \\ -5B_L B_s J_L^2 J_m k_s^2 N^2\omega + 5J_L^3 J_m k_s^3 N^2\omega + B_L^2 B_s J_L J_m k_s^2 N^2 + B_L B_m B_s J_L^2 k_s^2 N^2 + \\ +2B_L B_s^2 J_L J_m k_s^2 N^2 - B_L J_L^2 J_m k_s^3 N^2 - B_m J_L^3 k_s^3 N^2 - 2B_s J_L^2 J_m k_s^3 N^2 + \\ +B_L B_s^2 J_L^2 k_s^2 - B_s J_L^3 k_s^3) \end{pmatrix}}{(J_L^2 J_m K_m \mathbf{K}_p^i k_s^2 N^3 (B_L B_s - J_L k_s))}, \tag{15}$$

$$k_4 = -L_a \frac{\begin{pmatrix} -B_s J_L^4 J_m^2 N^4\omega^5 + 5J_L^4 J_m^2 k_s N^4\omega^4 - 10B_L J_L^3 J_m^2 k_s N^4\omega^3 + 10B_L^2 J_L^2 J_m^2 k_s N^4\omega^2 + \\ +10B_L B_s J_L^2 J_m^2 k_s N^4\omega^2 - 10J_L^3 J_m^2 k_s^2 N^4\omega^2 - 5B_L^3 J_L J_m^2 k_s N^4\omega - 10B_L^2 B_s J_L J_m^2 k_s N^4\omega - \\ -5B_L B_s^2 J_L J_m^2 k_s N^4\omega + 10B_L J_L^2 J_m^2 k_s^2 N^4\omega + 5B_s J_L^2 J_m^2 k_s^2 N^4\omega + B_L^4 J_m^2 k_s N^4 + \\ +3B_L^3 B_s J_m^2 k_s N^4 + 3B_L^2 B_s^2 J_m^2 k_s N^4 - 3B_L^2 J_L J_m^2 k_s^2 N^4 + B_L B_s^3 J_m^2 k_s N^4 - \\ -4B_L B_s J_L J_m^2 k_s^2 N^4 - B_s^2 J_L J_m^2 k_s^2 N^4 + J_L^2 J_m^2 k_s^3 N^4 - 5B_L B_s^2 J_L^2 J_m k_s N^2\omega + \\ +5B_s J_L^3 J_m k_s^2 N^2\omega + 2B_L^2 B_s^2 J_L J_m k_s N^2 + B_L B_m B_s^2 J_L^2 k_s N^2 + 2B_L B_s^3 J_L J_m k_s N^2 - \\ -3B_L B_s J_L^2 J_m k_s^2 N^2 - B_m B_s J_L^3 k_s^2 N^2 - 2B_s^2 J_L^2 J_m k_s^2 N^2 + J_L^3 J_m k_s^3 N^2 + \\ +B_L B_s^3 J_L^2 k_s - B_s^2 J_L^3 k_s^2 \end{pmatrix}}{(k_s J_L^2 J_m K_m \mathbf{K}_p^i N^3 (B_L B_s - J_L k_s)}, \tag{16}$$

$$k_5 = \omega^5 J_m N L_a J_L / (\mathbf{K}_p^i k_s K_m). \tag{17}$$

## 6. Results

To compare the system performance with synthesized controllers the value of the integral performance index and penalty function has been calculated.

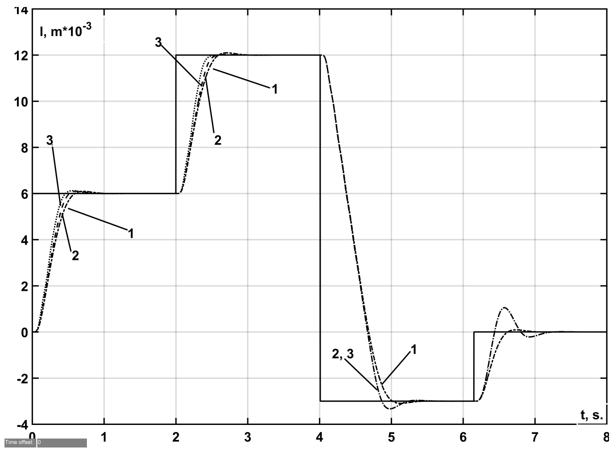
$$\begin{aligned} I^* &= I + F_{penalty} = \\ &= \left(\gamma_1 \int_0^T t\left|e\left(t\right)\right| dt\right) + \\ &+ \left(\gamma_2 \left(\frac{\omega_M(t)}{\omega_{M,\max}}\right)^2 H\left(\frac{\omega_M(t)}{\omega_{M,\max}}\right) + \right. \\ &\left. +\gamma_3 \left(\frac{v_{out}(t)}{v_{out,\max}}\right)^2 H\left(\frac{v_{out}(t)}{v_{out,\max}}\right)\right), \end{aligned} \tag{18}$$

where $I$ is classical integral performance index (see [21] and [22] for instance), $H(\cdot)$ is a Heaviside function, $\omega_{M,\max}$, $v_{out,\max}$ are specified maximum permissible overshoots and in the present case they make up 10 % and 5 % respectively. Coefficients $\gamma_i$, $i = \overline{1\ldots5}$ have been chosen for reasons of proportionality of the studied variables with each other.

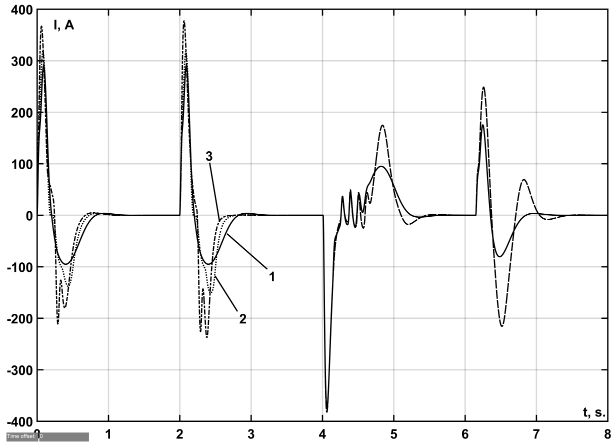
It should be noted that the value of the integral performance index was calculated for each segment of changes in the input signal separately: $I_i$ given $t \in [t_i, t_{i+1}]$.

The case of a system with two stable subsystems with different speed has been investigated in paper [23]. It suggested an approach to the formation of control inputs that would provide for the improvement of dynamic characteristics of mechatronic systems. The investigation for the cases of nonlinear systems with $J_L = const$ and the linear system with variable moment of inertia has been conducted in paper [8]. The obtained results show that the proposed approaches can ensure the desired behavior of the system and consider distinctive features of their models.

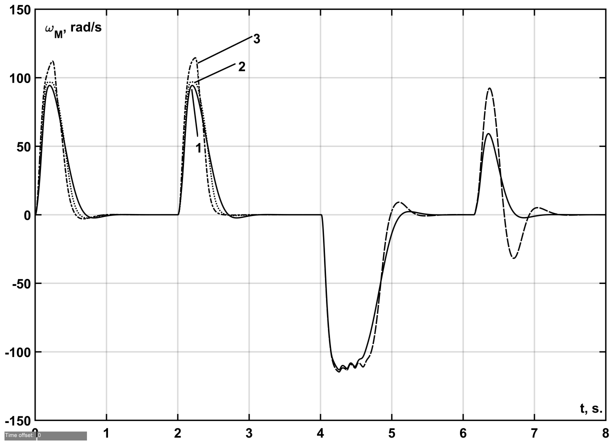
The quoted testify the feasibility of application of fuzzy logics for the design of nonlinear systems controller, since in this case the dynamic system characteristics are improved compared to the classic approach.



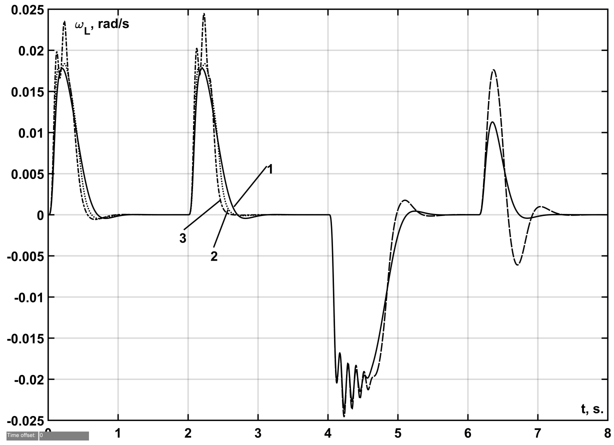

(a) System's output signal.




(b) Current in the motor armature.




(c) Angular velocity of the engine shaft rotation.




(d) Angular velocity of the second mass rotation.

**Fig. 4:** Results of the studied system simulation 1 - with traditional controller; 2 - system with subsystem that utilizes a standardBesselform 3 - system with unstable subsystem.

As evidenced by the results, the use of fuzzy controller allows for gains in the system speed in the absence of unacceptable overshooting. However, to achieve further gains in the speed, one can use an unstable subsystem in the domains of big deviations of the system's output coordinate. The investigation of these systems has been performed, in particular, in papers [13], [14], [15], [23] and [24].

In this case, an other fuzzy controller is added (highlighted with dotted line in Fig. 1). The output signal of this controller ensures smooth switching between the controller, which provides for an unstable behavior of the system, and the controller described in the above paragraphs.

State and output signals of the studied system are shown in Fig. 4. Corresponding performance indexes are given in Tab. 1. The given results state that the application of this approach does not result in inadmissible modes in the operation of the dynamic system, and increases the speed of transients.

**Tab. 1:** Performance indexes for systems with unstable subsystem.

| Type of controller | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $\boldsymbol{I}$ |
|---|---|---|---|---|---|
| Linear binominal | 215.8 | 212.1 | 10.05 | **0.5578** | 438.5 |
| Fuzzy Unstable Binominal | 8.1 | **1.044** | **9.638** | 0.8694 | **19.65** |
| Fuzzy Bessel Binominal | **7.8** | 1.318 | **9.638** | 0.9398 | 19.7 |

## 7. Conclusion

For synthesis of fuzzy controller of nonlinear system, it has been suggested to use the approach based on the dynamic systems family that takes into account nonlinearity of the plant. The conducted studies test if the applicability of this approach, which is based on the application of fuzzy set theory to the synthesis of nonlinear controllers. It has been also shown that the use of unstable subsystem allows improving the characteristics of the dynamic system without disrupting its stability.

## References


[1] OUNAS, D., M. RAMDANI, S. CHENIKHER and T. BOUKTIR. A combined methodology of fuzzy tracking control and birtual reference model for a PMSM. *Advances in Electrical and Electronic Engineering.* 2015, vol. 13, no. 3, pp. 212–222. ISSN 1336-1376. DOI: 10.15598/aeee.v13i3.1331.

[2] YONEYAMA, J. and Y. UCHIDA. Nonlinear control design based on generalized discrete-time Takagi-Sugeno fuzzy systems. In: *International conference on fuzzy systems (FUZZ).* Hyderabad: IEEE, 2013, pp. 1–6. ISSN 1098-7584. DOI: 10.1109/FUZZ-IEEE.2013.6622393.

[3] KHARRATI, H., S. KHANMOHAMMADI, W. PEDRYCZ and G. ALIZADEH. Improved polynomial fuzzy modeling and controller with stability analysis for nonlinear dynamical systems. *Mathematical problems in engineering.* 2012, vol. 2012, ID. 273631, pp. 1–21. ISSN 1024-123X. DOI: 10.1155/2012/273631.

[4] UANG, H.-J. and G. S. HUANG. A robust fuzzy model following observer-based control design for nonlinear System. In: *International conference on control applications.* Taipei: IEEE, 2004, pp. 171–176. ISBN 0-7803-8633-7. DOI: 10.1109/CCA.2004.1387206.

[5] SONG, C. and T. CHAI. A stable dual-mode fuzzy predictive control scheme for nonlinear system. In: *$5^{th}$ Asian Control Conference.* Melbourne: IEEE, 2004, pp. 1529–1536. ISBN 0-7803-8873-9. DOI: 10.1109/WCICA.2004.1340654.

[6] PEDRUKOVA, D. and P. FEDOR. A model-based fuzzy control of an induction motor. *Advances in Electrical and Electronic Engineering.* 2014, vol. 12, no. 5, pp. 427–434. ISSN 1336-1376. DOI: 10.15598/aeee.v12i5.1229.

[7] LOZYNSKYY, A. and L. DEMKIV. *Stability analysis of the system with Takagi-Sugeno controller.* 1st ed. Donetsk: IPAIMSU&NAS Ukraine, 2008, pp. 545–549. ISBN 978-3-642-24662-3.

[8] LOZYNSKYY, A. and L. DEMKIV. Application of fuzzy model at synthesis of automatic control system for nonlinear objects. *Elektromehanichni ta enerhozberihajuchi systemy.* 2015, vol. 2, iss. 30, pp. 24–30. ISSN 2072-2052.

[9] HOSTEV, V. I. *Fuzzy controllers in automation control systems.* 1st ed. Kyiv: Radioamator, 2008. ISBN 978-966-96178-2-0.

[10] TAKAGI, T. and M. SUGENO. Fuzzy identification of systems and its applications to modeling and control. *IEEE Transactions on systems, man, and cybernetics.* 1985, vol. 15, no. 1, pp. 116–132. ISSN 0018-9472. DOI: 10.1109/TSMC.1985.6313399.

[11] PIEGAT, A. *Modelowanie i sterowanie rozmyte.* 1st ed. Warszawa: Akademicka Oficyna Wydawnicza EXIT, 203. ISBN 83-87674-14-1.

[12] LOZYNSKYY, A. and L. DEMKIV. Relatively stable system with fuzzy controller, artificial intelligence. *Scientific-theoretical magazine "Artificial intelligence".* 2010, vol. 2010, iss. 3, pp. 415–420. ISSN 1561-5367.

[13] LOZYNSKYY, A. and L. DEMKIV. The Influence of Membership Function's Parameters of Fuzzy Controller, with Established Unstable Subsystem, on Example of Two-mass Electromechanical System. *Elektrotechnika ta elektroenerhetyka.* 2012, vol. 89, no. 2a/2013, pp. 4–11. ISSN 1607-6761.

[14] LOZYNSKYY, A. and L. DEMKIV. Forming of controlled influences in the system with fuzzy regulator. *Computational problems of electrical engineering.* 2011, vol. 1, iss. 1, pp. 27–34. ISSN 2224-0977.

[15] LOZYNSKYY, A. and L. DEMKIV. Investigation of multicriteria optimal control with time-variable weight Coefficients. *Przeglad Elektrotechniczny.* 2012, vol. 2012, no. 2a, pp. 195–198. ISSN 0033-2097.

[16] OSICHEV, A. V., V. O. KOTLJAROV and V. S. MARKOV. Standard distribution of roots in the synthesis of the problems in the drive. In: *Problemy avtomatyzyrovannoho elektropryvoda.* Kharkiv: Osnova, 1997, pp. 104–109. ISBN 5-7768-0313-6.

[17] MARUSHCHACK, Y. Y. *Synthesis of electromechanical systems with serial and parallel adjustment.* 1st ed. Lviv: Vydavnytstvo NU "Lvivska politekhnika", 2005. ISBN 966-553-424-6.

[18] KARA, T. and I. EKER. Nonlinear modeling and identification of a DC motor for bidirectional operation with real time experiments. *Energy conversion and management.* 2004, vol. 45, iss. 7–8, pp. 1087–1106. ISSN 0196-8904. DOI: 10.1016/j.enconman.2003.08.005.

[19] MARUSHCHACK, Y. Y., A. LOZYNSKYI and A. P. KUSHNIR. *Dynamics of two-mass stabilization mode in electric arc furnaces.* 1st ed. Lviv: NU "Lvivska politekhnika", 2011. ISBN 978-617-607-145-7.

[20] LOZYNSKYI, A., Y. PARAHCNUK and L. DEMKIV. Investigation of the translation of electrode arc furnace with a fuzzy controller. *Tekhnichna Elektrodynamika.* 2014, vol. 2, iss. 1, pp. 73–77. ISSN 1607-7970.

[21] NAGRATH, I. J. and M. GOPAL. *Control systems engineering.* 5th ed. New Delhi: New Age International Publisher, 2009. ISBN 978-8122420081.

[22] DUARTE-MERMOUD, M. A. and R. A. PRIETO. Performance index for quality response of dynamical systems. *ISA Transactions.* 2004, vol. 43, iss. 1, pp. 133–151. ISSN 0019-0578. DOI: 10.1016/S0019-0578(07)60026-3.

[23] LOZYNSKYY, A. and L. DEMKIV. Investigation of systems with unstable subsystem. *Elektromekhanika ta elektroenerhetyka.* 2010, vol. 1, iss. 1, pp. 19–29. ISSN 1607-6761.

[24] DEMKIV, L. The influence of parameters of membership functions on quality performance of the system with two roots in the right half-plane. *Vesnik of the National University.* 2012, vol. 46, no. 673, pp. 39–45. ISSN 0321-0499.

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# Appendix A - System Parameters

- $R_a = 0.5\ \Omega$,
- $L_a = 0.021{\cdot}0.5$ H,
- $J_m = 0.8325$ kg·m$^2$,
- $J_L = 77400.2$ kg·m$^2$,
- $k_s = 24{\cdot}10^7$.
- $N = 5375$.
- $B_L = B_M = 0$.