TOTAL EDDY CURRENTS INDUCED IN SCREENS OF A SYMMETRICAL THREE-PHASE SINGLE-POLE GAS-INSULATED TRANSMISSION LINE (GIL)

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Summary: In the paper we discuss the question of eddy currents induced in screens of a symmetrical three-phase single-pole gas-insulated transmission line (GIL). First, we determine the eddy currents induced in the tubular screen by the magnetic field of self-current of the phase conductor. Then the magnetic field in the external parallel phase conductor is presented by means of a vector magnetic potential as Fourrier series. In the non-conducting external and internal area of the screen we use Laplace equation for the magnetic field strength taking into account the reverse reaction of eddy currents induced in the screen. In the conducting screen we apply Helmholtz equation for eddy currents density. Using classical boundary conditions we determine the density of the currents. The solutions obtained are used to determine the total eddy currents induced in all the screens of the GIL under consideration.

1. INTRODUCTION

Single-pole transmission lines with isolated phases are built for high and the highest voltages. Each phase is placed in a separate sheath (IPGIL - **I**solated **P**hase **G**as **I**nsulated **L**ine). A phase conductor is usually a tubular or molded conductor made of aluminum, aluminum alloy or copper alloy. One of the ways of laying such GILs is to place them at the apexes of an equilateral triangle- it is the so-called symmetrical system (fig.1) [1]-[12].

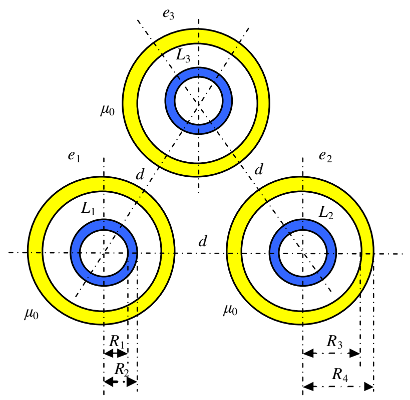

Fig.1. Symmetrical transmission line with insulated phases

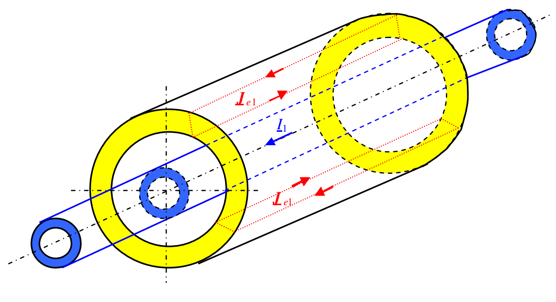

Fig.2. Eddy currents induced in the screen by the magnetic field of the phase conductor self-current

In order to determine the self- and mutual impedances of GIL, the power losses, the heating up of the screens and the forces occurring in case of a short-circuit it is necessary to calculate the eddy currents induced in the screen by the magnetic field of the current of the screened conductor (fig.2), as well as of the currents of the neighboring phases (fig.3).

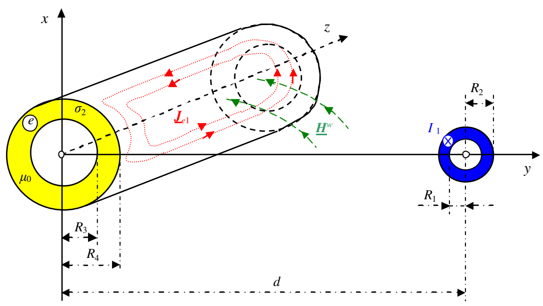

Fig. 3. Eddy currents induced in the screen by the magnetic field of the neighboring phase conductor current

The currents induced in the screen by the magnetic field of the currents in phase conductors of neighboring phases, when the distance between the screens is long enough, are calculated approximately assuming that the screen under consideration is in uniform magnetic field created by the currents in the conductors of neighboring phases. In the case of the size of the two screens being close to the distance between them the assumption of the uniformity of the field is a huge simplification, one has to expect the influence of the proximity effect on the distribution of eddy currents in the screen. In the above case the determination of these eddy currents in the screens of a three-phase single-pole GIL can be done using the method of eddy currents projection described in reference [16] and quoted in [15]. However, this method is limited to the so-called thin walled screens. In references [17], [18] another method of calculating eddy currents induced in the tubular screen of GIL is presented, using the development into Fourier series of the external magnetic field components in relation to the screen under consideration. To determine these currents one can also apply numerical methods [19] or analytical-numerical ones [8]-[12]. In this paper we propose a method of calculating eddy currents in all the screens of a flat three-phase single-pole GIL using the vector magnetic potential.

2. EDDY CURRENTS IN A SYSTEM OF TWO TUBULAR CO-AXIAL PARALLEL CONDUCTORS

Let us consider the proximity effect in a system of a tubular conductor with conductivity σ_1, internal radius R_1 and external R_2 with sinusoidal current of compound efficient quantity $\underline{I}_1$, screened with a co-axial tubular conductor with conductivity σ_2, internal radius R_3 and external R_4.

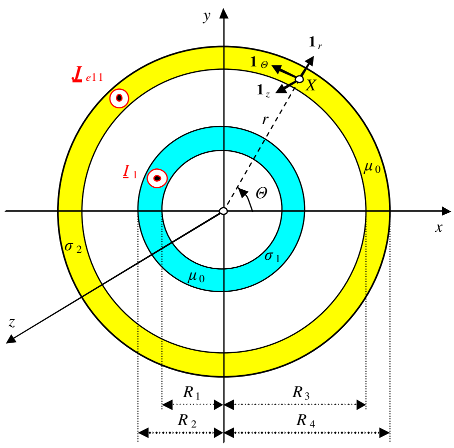

Fig.4. Tubular conductor with current $\underline{I}_1$ screened with a co-axial tubular screen

The induced current density $\underline{\boldsymbol{J}}_{e11}(r) = \mathbf{1}_z \underline{J}_{e11}(r)$ in the screens has one component, it is a function of one variable r and it fulfills the modified Bessel equation

$$\frac{\mathrm{d}^2 \underline{J}_{e11}(r)}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\underline{J}_{e11}(r)}{\mathrm{d}r} - \underline{\Gamma}^2 \underline{J}_{e11}(r) = 0 \qquad (1)$$

where $\underline{\Gamma}_2 = \sqrt{\mathrm{j}\,\omega\mu_0\sigma_2} = \sqrt{2\mathrm{j}}\,k_2 = \sqrt{2\mathrm{j}}\,\frac{1}{\delta_2}$ is the complex diffusion constant where the depth of the electromagnetic wave penetration into the screen $\delta_2 = \frac{1}{k_2} = \sqrt{\frac{2}{\omega\mu_0\sigma_2}}$. The solution of Eq. (1) is [10], [12], [14]

$$\underline{J}_{e11}(r) = \frac{\underline{\Gamma}_2 \underline{I}_1}{2\pi R_3}\,\frac{\underline{b}\,I_0(\underline{\Gamma}_2 r) + \underline{c}\,K_0(\underline{\Gamma}_2 r)}{\underline{d}} \qquad (2)$$

where $I_0(\underline{\Gamma}_2 r)$ and $K_0(\underline{\Gamma}_2 r)$ stands for the modified Bessel functions of zero order, of the first and second kind respectively. For $\beta = \frac{R_3}{R_4}$ ($0 \le \beta \le 1$)

$$\underline{d} = I_1(\underline{\Gamma}_2 R_4)\,K_1(\underline{\Gamma}_2 R_3) - I_1(\underline{\Gamma}_2 R_3)\,K_1(\underline{\Gamma}_2 R_4) \qquad (2a)$$

$$\underline{b} = \beta\,K_1(\underline{\Gamma}_2 R_3) - K_1(\underline{\Gamma}_2 R_4) \qquad (2b)$$

and

$$\underline{c} = \beta\,I_1(\underline{\Gamma}_2 R_3) - I_1(\underline{\Gamma}_2 R_4) \qquad (2c)$$

The frequency of sinusoidally alternating current and the conductivity of the screen in relation to its external radius will be taken into account due to the coefficient $\alpha = \frac{R_2}{\delta_2} = k_2\,R_2$. Thus we can also introduce a non-dimensional variable $\xi = \frac{r}{R_2}$, $\beta \le \xi \le 1$, and then express the current density in the tubular conductor as a function of the non-dimensional variable and refer it to the current density

$$\underline{J}_{01} = \frac{\underline{I}_1}{\pi\,(R_4^2 - R_3^2)} \qquad (3)$$

obtaining the distribution of the density of current induced in the tubular screen as a relative quantity

$$\underline{j}_{e11}(\xi) = \frac{\sqrt{2\mathrm{j}}\,\alpha}{2}\,\frac{1-\beta^2}{\beta}\,\frac{\underline{b}\,I_0(\sqrt{2\mathrm{j}}\,\alpha\xi) + \underline{c}\,K_0(\sqrt{2\mathrm{j}}\,\alpha\xi)}{\underline{d}} \qquad (4)$$

3. EDDY CURRENTS IN A SYSTEM OF TWO PARALLEL TUBULAR CONDUCTORS

Let us consider the proximity effect in a system of a tubular conductor with conductivity σ_2, internal radius R_3 and external R_4 parallel to an external tubular conductor with conductivity σ_1, internal radius R_1 and external R_2 with sinusoidal current of efficient compound quantity $\underline{I}_1$ (fig. 5).

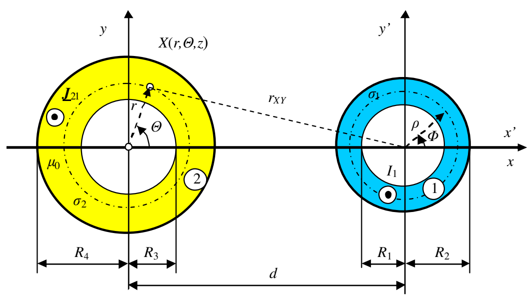

Fig. 5. Tubular conductor in non-uniform magnetic field of current $\underline{I}_1$ in the external tubular conductor

Foucault phenomenon in the system shown in figure 5 consists of inducing, in the conductor, the eddy currents $\underline{\boldsymbol{J}}_{21}(r,\Theta)=\mathbf{1}_z\underline{J}_{21}(r,\Theta)$ by sinusoidally time varying magnetic field $\underline{\boldsymbol{H}}^w(r_{XY})$, generated by current $\underline{I}_1$ in the second conductor.

The vector magnetic potential generated by current $\underline{I}_1$ has only one component along Oz axis and it is the source potential in relation to the first conductor, so $\underline{\boldsymbol{A}}^w(r_{XY})=\mathbf{1}_z\underline{A}^w(r_{XY})$ and according to its definition in a cylindrical system of coordinates (ρ,Φ,z) connected with the second conductor, we have [12] for $r<d$

$$\underline{A}^w(r,\Theta)=\frac{\mu_0\underline{I}_1}{2\pi}\left[\ln\frac{1}{d}+\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{r}{d}\right)^n\cos n\Theta\right]+\underline{A}_0 \qquad (5)$$

where the constant $\underline{A}_0$ can be freely assumed.

The magnetic field strength vector is calculated from the definition of the vector magnetic potential

$$\underline{\boldsymbol{H}}^w(r,\Theta)=\mathbf{1}_r\underline{H}_r^w(r,\Theta)+\mathbf{1}_\Theta\underline{H}_\Theta^w(r,\Theta) \qquad (6)$$

where

$$\underline{H}_r^w(r,\Theta)=-\frac{\underline{I}_1}{2\pi r}\sum_{n=1}^{\infty}\left(\frac{r}{d}\right)^n\sin n\Theta \qquad (6a)$$

and

$$\underline{H}_\Theta^w(r,\Theta)=-\frac{\underline{I}_1}{2\pi r}\sum_{n=1}^{\infty}\left(\frac{r}{d}\right)^n\cos n\Theta \qquad (6b)$$

Magnetic field $\underline{\boldsymbol{H}}^{ext}(r,\Theta)$ in the external area ($r\ge R_4$) of the first conductor is expressed by the formula

$$\underline{\boldsymbol{H}}^{ext}=\underline{\boldsymbol{H}}^w+\underline{\boldsymbol{H}}^{rr} \qquad (7)$$

where $\underline{\boldsymbol{H}}^{rr}(r,\Theta)$ is the reverse reaction magnetic field, created by eddy currents $\underline{\boldsymbol{J}}_{21}(r,\Theta)=\mathbf{1}_z\underline{J}_{21}(r,\Theta)$, generated in the considered conductor by the magnetic field $\underline{\boldsymbol{H}}^w(r,\Theta)$.

The electric field strength $\underline{\boldsymbol{E}}^{rr}(r,\Theta)$ accompanying the magnetic field $\underline{\boldsymbol{H}}^{rr}(r,\Theta)$ in this area has one component, thus $\underline{\boldsymbol{E}}^{rr}(r,\Theta)=\mathbf{1}_z\underline{E}^{rr}(r,\Theta)$. Then this field fulfills the scalar Laplace equation

$$\nabla^2\underline{E}^{rr}(r,\Theta)=0 \qquad (8)$$

A general solution of equation (8) is [12],[17] and [18]

$$\underline{E}^{rr}(r,\Theta)=\sum_{n=1}^{\infty}\underline{E}_n^{rr}(r,\Theta)=\sum_{n=1}^{\infty}\underline{B}_n\frac{1}{r^n}\cos n\Theta \qquad (8a)$$

where $\underline{B}_n$ is a constant, which will be calculated later from boundary conditions.

From the second Maxwell equation we obtain the reverse reaction magnetic field

$$\underline{\boldsymbol{H}}^{rr}(r,\Theta)=\mathbf{1}_r\underline{H}_r^{rr}(r,\Theta)+\mathbf{1}_\Theta\underline{H}_\Theta^{rr}(r,\Theta) \qquad (9)$$

where

$$\underline{H}_r^{rr}(r,\Theta)=\sum_{n=1}^{\infty}\frac{n\underline{B}_n}{\mathrm{j}\omega\mu_0 r^{n+1}}\sin n\Theta \qquad (9a)$$

and

$$\underline{H}_\Theta^{rr}(r,\Theta)=-\sum_{n=1}^{\infty}\frac{n\underline{B}_n}{\mathrm{j}\omega\mu_0 r^{n+1}}\cos n\Theta \qquad (9b)$$

The above considerations let us present the magnetic field $\underline{\boldsymbol{H}}^{ext}(r,\Theta)$ in external area ($r\ge R_4$) of the first conductor in the following form:

$$\underline{\boldsymbol{H}}^{ext}(r,\Theta)=\mathbf{1}_r\underline{H}_r^{ext}(r,\Theta)+\mathbf{1}_\Theta\underline{H}_\Theta^{ext}(r,\Theta) \qquad (10)$$

where

$$\underline{H}_r^{ext}(r,\Theta)=-\sum_{n=1}^{\infty}\left[\frac{\underline{I}_1}{2\pi r}\left(\frac{r}{d}\right)^n-\frac{n\underline{B}_n}{\mathrm{j}\omega\mu_0 r^{n+1}}\right]\sin n\Theta \qquad (10a)$$

and

$$\underline{H}_\Theta^{ext}(r,\Theta)=-\sum_{n=1}^{\infty}\left[\frac{\underline{I}_1}{2\pi r}\left(\frac{r}{d}\right)^n+\frac{n\underline{B}_n}{\mathrm{j}\omega\mu_0 r^{n+1}}\right]\cos n\Theta \qquad (10b)$$

In the conductor ($R_3\le r\le R_4$) the eddy current density $\underline{\boldsymbol{J}}_{21}(r,\Theta)=\mathbf{1}_z\underline{J}_{21}(r,\Theta)$ fulfills the scalar Helmholtz equation

$$\nabla^2\underline{J}_{21}(r,\Theta)=\underline{\Gamma}_2^2\underline{J}_{21}(r,\Theta) \qquad (11)$$

Applying the separation of variables method in cylindrical coordinates, we look for the solution of equation (11) in the following form [12]

$$\underline{J}_{21}(r,\Theta)=\sum_{n=1}^{\infty}\left[\underline{C}_n I_n(\underline{\Gamma}_2 r)+\underline{D}_n K_n(\underline{\Gamma}_2 r)\right]\cos n\Theta \qquad (11a)$$

From the second Maxwell equation we obtain[1] the magnetic field in the considered conductor

$$\underline{\boldsymbol{H}}_{21}(r,\Theta)=\mathbf{1}_r\underline{H}_{21r}(r,\Theta)+\mathbf{1}_\Theta\underline{H}_{21\Theta}(r,\Theta) \qquad (12)$$

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[1] The formulas (165) and (173) on page 281 in reference [22] let us derive $\frac{\mathrm{d}I_n(\underline{\Gamma}r)}{\mathrm{d}r}=-\frac{n}{r}I_n(\underline{\Gamma}r)+\underline{\Gamma}I_{n-1}(\underline{\Gamma}r)$ and on the basis of formula (167) on page 281 we also get $2nI_n(\underline{\Gamma}r)=\underline{\Gamma}rI_{n-1}(\underline{\Gamma}r)-\underline{\Gamma}rI_{n+1}(\underline{\Gamma}r)$. Formulas (210) and (218) on page 285 let us derive $\frac{\mathrm{d}K_n(\underline{\Gamma}r)}{\mathrm{d}r}=-\frac{n}{r}K_n(\underline{\Gamma}r)-\underline{\Gamma}K_{n-1}(\underline{\Gamma}r)$ and on the basis of formula (210) on page 281 we also get $2nK_n(\underline{\Gamma}r)=\underline{\Gamma}rK_{n+1}(\underline{\Gamma}r)-\underline{\Gamma}rK_{n-1}(\underline{\Gamma}r)$.

where

$$\underline{H}_{21r}(r,\Theta)= \\ =\frac{1}{\underline{\Gamma}_1^2 r}\sum_{n=1}^{\infty} n\left[\underline{C}_n\, I_n(\underline{\Gamma}_2 r)+\underline{D}_n\, K_n(\underline{\Gamma}_2 r)\right]\sin n\Theta \quad (12a)$$

and

$$\underline{H}_{21\Theta}(r,\Theta)=-\frac{1}{\underline{\Gamma}_1^2 r}\sum\left\{\underline{C}_n\left[n I_n(\underline{\Gamma}_2 r)-\underline{\Gamma}_2 r\, I_{n-1}(\underline{\Gamma}_2 r)\right]+ \right. \\ \left. +\underline{D}_n\left[n K_n(\underline{\Gamma}_2 r)+\underline{\Gamma}_1 r\, K_{n-1}(\underline{\Gamma}_2 r)\right]\right\}\cos n\Theta \quad (12b)$$

Inside the considered conductor, i.e. for $0 \le r \le R_3$, the electric field has one component $\underline{\boldsymbol{E}}^{int}(r,\Theta)=\mathbf{1}_z \underline{E}^{int}(r,\Theta)$ fulfilling the scalar Laplace equation (8), whose solution obtained using the separation of variables method, has the form [12]

$$\underline{E}^{int}(r,\Theta)=\sum_{n=1}^{\infty}\underline{F}_n\, r^n \cos n\Theta \quad (13)$$

where $\underline{F}_n$ is a constant, which will be calculated later from boundary conditions.
Applying the second Maxwell equation to formula (13), we obtain the compound form of the vector of the magnetic field strength inside the tubular conductor

$$\underline{\boldsymbol{H}}^{int}(r,\Theta)=\sum_{n=1}^{\infty}\frac{n\underline{F}_n\, r^{n-1}}{\mathrm{j}\,\omega\mu_0}\left(\mathbf{1}_r \sin n\Theta+\mathbf{1}_\Theta \cos n\Theta\right) \quad (14)$$

If we assume that the magnetic permittivity of the considered conductor $\mu_1=\mu_0$, we can write down the following boundary conditions for the magnetic field:

- for $r=R_3$

$$\underline{\boldsymbol{H}}^{int}(r=R_3,\Theta)=\underline{\boldsymbol{H}}_{12}(r=R_3,\Theta) \quad (15)$$

- for $r=R_4$

$$\underline{\boldsymbol{H}}^{ext}(r=R_4,\Theta)=\underline{\boldsymbol{H}}_{12}(r=R_4,\Theta) \quad (15a)$$

The above conditions imply the equality of tangent and ordinary components of the magnetic field on the border of two mediums. Thus we obtain a system of four equations with unknown quantities $\underline{B}_n$, $\underline{C}_n$, $\underline{D}_n$ and $\underline{F}_n$.

After the determination of the constants $\underline{C}_n$ and $\underline{D}_n$ we obtain a formula for the current density in the tubular conductor in the following form

$$\underline{J}_{21}(r,\Theta)=-\frac{\underline{\Gamma}_2 \underline{I}_1}{\pi R_4}\sum_{n=1}^{\infty}\left(\frac{R_4}{d}\right)^n \underline{f}_n(r)\cos n\Theta \quad (16)$$

where the function

$$\underline{f}_n(r)=\frac{K_{n+1}(\underline{\Gamma}_2 R_3)\, I_n(\underline{\Gamma}_2 r)+I_{n+1}(\underline{\Gamma}_2 R_3)\, K_n(\underline{\Gamma}_2 r)}{I_{n-1}(\underline{\Gamma}_2 R_4)\, K_{n+1}(\underline{\Gamma}_2 R_3)-I_{n+1}(\underline{\Gamma}_2 R_3)\, K_{n-1}(\underline{\Gamma}_2 R_4)} \quad (16a)$$

in which I_n are I_{n+1} modified Bessel functions of the first kind while K_n are K_{n+1} are modified Bessel functions of the second kind of n and $n+1$ order respectively.
If we introduce a relative quantity of the distance between two conductors $\lambda=\frac{d}{R_4}\ge 1$ and relate current density (16) to the quantity (3), we will get the distribution of densities of the current induced in the tubular conductor as a relative quantity

$$\underline{j}_{21}(\xi,\Theta)=-\sqrt{2}\mathrm{j}\alpha\left(1-\beta^2\right)\sum_{n=1}^{\infty}\left(\frac{1}{\lambda}\right)^n \underline{f}_n(\xi)\cos n\Theta \quad (17)$$

If the conductor with current $\underline{I}_1$ is to the left of the considered tubular conductor we obtain the density of the induced current

$$\underline{J}_{21}(r,\Theta)=-\frac{\underline{\Gamma}_2 \underline{I}_1}{\pi R_4}\sum_{n=1}^{\infty}(-1)^n\left(\frac{R_4}{d}\right)^n \underline{f}_n(r)\cos n\Theta \quad (18)$$

and its relative quantity

$$\underline{j}_{21}(\xi,\Theta)=-\sqrt{2}\mathrm{j}\alpha\left(1-\beta^2\right)\sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{\lambda}\right)^n \underline{f}_n(\xi)\cos n\Theta \quad (18a)$$

4. TOTAL EDDY CURRENTS IN THE SCREENS OF A SYMMETRICAL THREE-PHASE GIL

Total current denisty in the screens of the considered transmission line is a sum of the relevant densities of currents induced by the magnetic fields of all the three phase currents.
For the screen of phase L_1 then

$$\underline{J}_{e1}(r,\Theta)=\underline{J}_{e11}(r,\Theta)+\underline{J}_{e12}(r,\Theta)+\underline{J}_{e13}(r,\Theta) \quad (19)$$

where $\underline{J}_{e11}(r,\Theta)$ is defined by the formula (2) and

$$\underline{J}_{e12}(r,\Theta)=-\frac{\underline{\Gamma}_2 \underline{I}_2}{\pi R_4}\sum_{n=1}^{\infty}\left(\frac{R_4}{d}\right)^n \underline{f}_n(r)\cos n\Theta \quad (19a)$$

and

$$\underline{J}_{e13}(r,\Theta)=-\frac{\underline{\Gamma}_2 \underline{I}_3}{\pi R_4}\sum_{n=1}^{\infty}\left(\frac{R_4}{2d}\right)^n \underline{f}_n(r)\cos n\Theta \quad (19b)$$

Then, current density $\underline{J}_{e1}(r,\Theta)$ depends on currents $\underline{I}_1$, $\underline{I}_2$ and $\underline{I}_3$. The relative quantity of this density

$$\underline{j}_{e1}(\xi,\Theta)=\frac{\underline{J}_{e1}(\xi,\Theta)}{\underline{J}_{01}} \tag{20}$$

is defined with reference to the mutual relation between these currents. If they form a symmetrical three of a three-phase system currents, i.e. when $\underline{I}_2=\exp[-\mathrm{j}\frac{2}{3}\pi]\underline{I}_1$ and $\underline{I}_3=\exp[\mathrm{j}\frac{2}{3}\pi]\underline{I}_1$ the relative quantity referring to current $\underline{I}_2$ is defined by the formula

$$\begin{aligned}\underline{j}_{e12}(\xi,\Theta)&=-\sqrt{2}\mathrm{j}\alpha\left(1-\beta^2\right)\times\\&\times\exp[-\mathrm{j}\frac{2}{3}\pi]\sum_{n=1}^{\infty}\left(\frac{1}{\lambda}\right)^n\underline{f}_n(\xi)\cos n\Theta\end{aligned} \tag{21}$$

and the relative quantity referring to current $\underline{I}_3$ is defined by the formula

$$\begin{aligned}\underline{j}_{e13}(\xi,\Theta)&=-\sqrt{2}\mathrm{j}\alpha\left(1-\beta^2\right)\times\\&\times\exp[\mathrm{j}\frac{2}{3}\pi]\sum_{n=1}^{\infty}\left(\frac{1}{2\lambda}\right)^n\underline{f}_n(\xi)\cos n\Theta\end{aligned} \tag{22}$$

Density J_{e2} of current induced in screen e_2 is a sum

$$\underline{J}_{e2}(r,\Theta)=\underline{J}_{e22}(r,\Theta)+\underline{J}_{e21}(r,\Theta)+\underline{J}_{e23}(r,\Theta) \tag{23}$$

Current density $\underline{J}_{e22}(r,\Theta)$ is defined by the formula (2). Current density $\underline{J}_{e21}(r,\Theta)$ is defined by the formula (18). Current density $\underline{J}_{e23}(r,\Theta)$ is given by the formula (16), where current $\underline{I}_1$ should be replaced with current $\underline{I}_3$.

Density J_{e3} of the current induced in screen e_3 is a sum of density J_{e33} induced by the magnetic field of current $\underline{I}_3$, density J_{e31} of the current induced by the magnetic field of current $\underline{I}_1$ and of density J_{e32} of the current induced by the magnetic field of current $\underline{I}_2$ in the neighboring phase conductors, thus

$$\underline{J}_{e3}(r,\Theta)=\underline{J}_{e33}(r,\Theta)+\underline{J}_{e31}(r,\Theta)+\underline{J}_{e32}(r,\Theta) \tag{24}$$

Current density $\underline{J}_{e33}(r,\Theta)$ is defined by the formula (2). Current density $\underline{J}_{e31}(r,\Theta)$ is defined by the formula (18) after replacing the distance d with $2d$. Current density $\underline{J}_{e32}(r,\Theta)$ is defined by the same formula, where current $\underline{I}_1$ is replaced with current $\underline{I}_2$.

The distribution of modules of the above densities of total eddy currents induced in the screens of a symmetrical three-phase transmission line with insulated phases is shown in figure 6.

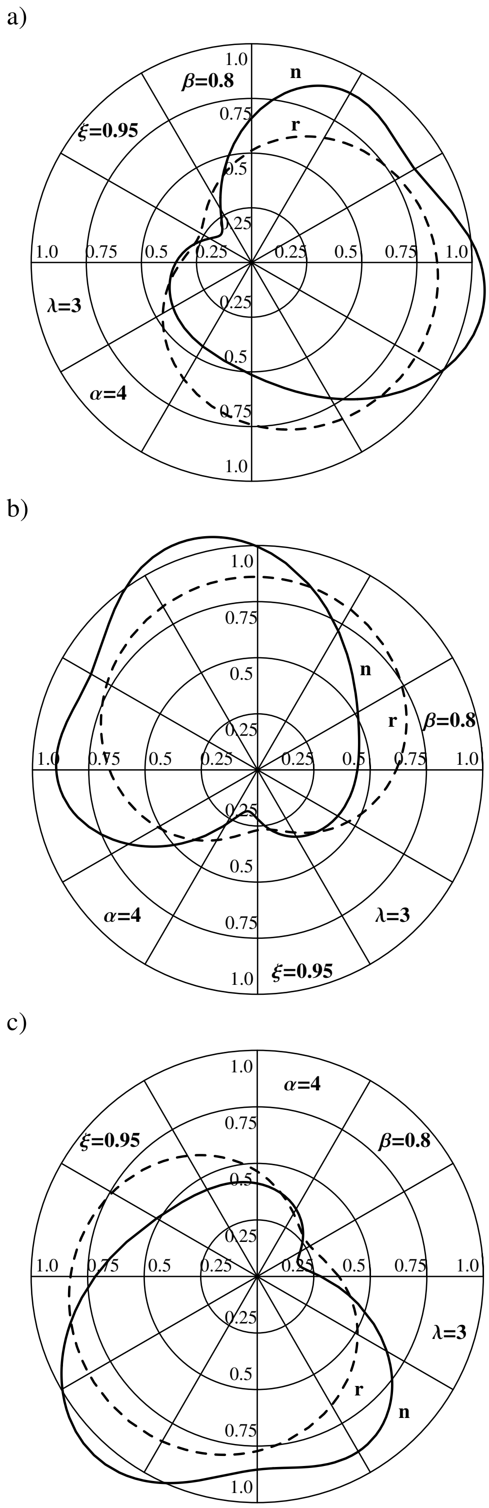

*Fig. 6. Distribution of modules of total current densities induced in the screens of a symmetrical three-phase single-pole GIL: a) in screen e_1, b) in screen e_2, c) in screen e_3; **n** – non-uniform magnetic field, **r** – uniform magnetic field*

5. CONCLUSIONS

Taking into account the so-called reverse reaction of eddy currents induced in screens of transmission lines allows, together with the application of Laplace and Helmholtz equations, the calculation of these currents densities in the form of analytical formulas expressed by Bessel function. One can use these formulas in the range of frequencies enabling the omission of the shift currents and they refer to screens with any electrical and geometrical parameters, including the thickness of their walls.

The numerical calculations we carried out (fig. 6) lead to the conclusion that the distributions of the densities of currents in the screens of a symmetrical GIL differ from one another and are not symmetrical in spite of the symmetrical three of currents and of the symmetrical geometrical configuration. Some symmetry of induced eddy currents occurs only for $n = 1$. It can be essential in the determination of power losses and temperature distribution in the screens and in the determination of electrodynamics forces between the screens.

Using the formula for the current density in the screen (tubular conductor) one can also determine the power losses in the screen. If moreover we determine the constant $\underline{B}_n$ appearing in equations (10a) and (10b), we can determine the magnetic field outside the screen as well as the reverse reaction magnetic field, hence also the coefficient of this reverse reaction. The calculation of the constant $\underline{F}_n$ appearing in equation (14), lets us also determine the magnetic field outside the screen and then the coefficient of the reverse reaction.

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