VARIATION OF MAGNETIC FLUX ON THE OCCURRENCE OF FERRORESONANCE IN A TRANSFORMER USING FINITE ELEMENT METHOD

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Abstract. *Ferroresonance is an oscillation in a non-linear circuit containing transformers at light load or no load coupled with a capacitor. The non-linear behavior of the core material of the transformer at no load is represented by a nonlinear equation. The capacitive effects of the power network are treated as a capacitor in series with the transformer equivalent circuit. As the system contains a nonlinear equation, it was difficult to analyze the model using the linear method. The finite Element Method is used here for the simulations of ferroresonance. The Finite Element Method allows a field solution to be obtained, even with time-varying fields with non-homogeneous or non-linear fields. This method helps to find out the changes that occur in the magnetic structure of the core under ferroresonance. Supply voltage is taken as the variable parameter. Capacitor and transformer voltages along with the phase plane are analyzed before and after the occurrence of ferroresonance over-voltage. Distribution and change in the magnetic flux during the triggering of ferroresonance are also examined.*

Keywords

Ferroresonance, Over Voltage, Finite element method, Magnetic flux density, non-linear model of transformer.

1. Introduction

Ferroresonance in a power system leads to sustained over-voltages across the transformers and the line capacitance [1] and over-current through the transformer coils. The magnetic behavior of the core of a transformer under saturation introduces the nonlinearity in the system. Ferroresonance may appear in a lightly loaded transformer, as a result of the opening of transformer phases, switching operations, load shedding, lightning surges [2], removal of the fault [3], or sharp load change [4]. Thus a ferroresonance model can be built up with a power transformer at no load conditions along with a capacitor in series as shown in Fig. 1. This model was proposed by R. Rudenberg in his literature [5].

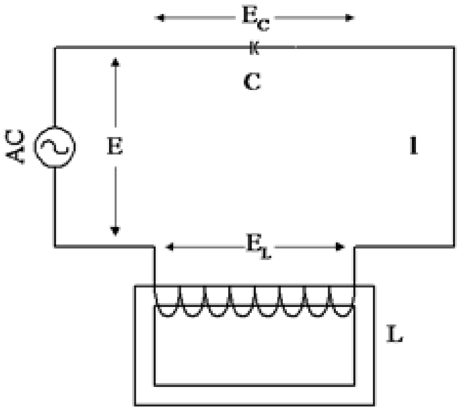


Fig. 1: Ferroresonance series circuit.

Laboratory test data is used for the modeling of the transformer. Then the nonlinear mathematical equations for the system shown in Fig. 1 are developed to study ferroresonance.

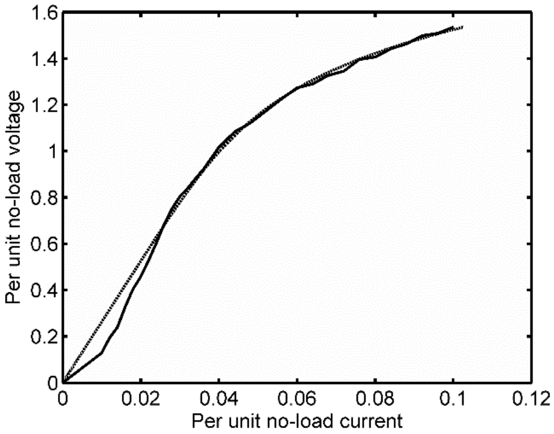


Fig. 2: Experimental (solid) and Simulated (dotted) Current vs. Voltage of a transformer at no load.

R. Rudenberg in his study [5] shows the effect of circuit parameters like supply voltage, supply frequency, and series capacitance on the occurrence of ferroresonance. The recent literature mainly studies the two aspects of ferroresonance. One is the detection and mitigation of ferroresonance [6–8]. And other is the study of the impact on the system under ferroresonance [9–11]. This paper studies the effect on the magnetic flux of the transformer core when the ferroresonance occurs.

The requirement for more accurate results in the process of design and analysis of the system invited the spreading of numerical methods suitable for computing electric and magnetic fields. While an analytical solution is difficult to achieve because of the complex geometry and nonlinearity, in most cases only a numerical solution is possible. The finite element method may be suitable for this purpose.

An effective computing method for the circuit-field transients has been proposed in [12]. Here the finite element method is used to simulate the field part. Authors in [13] show the detailed effect of ferroresonance on the transformer windings. Here the electromagnetic transient solution is merged along with the finite element field solution. In the work [14] authors study the effect of amorphous steels on the anti-resonance properties of the core.

2. The Transformer Model

To develop a ferroresonance study circuit, at first, a suitable transformer model is required. However, the non-linearity of the core material of the transformer makes it the most critical part of the study. One of the methods is the Jiles-Atherton method. This method is easy to implement in the software and shows high accuracy in approximating the hysteresis loop [15]. Duality-based approaches namely, "the reluctance-based" and "the direct application" for developing the low-frequency transformer models have been discussed in [16]. Paper [17] uses a laboratory setup to record instantaneous voltage and current against time. The data of magnetic flux was obtained by connecting a capacitor in parallel to the transformer and the transformer magnetizing current from a small resistor connected in series.

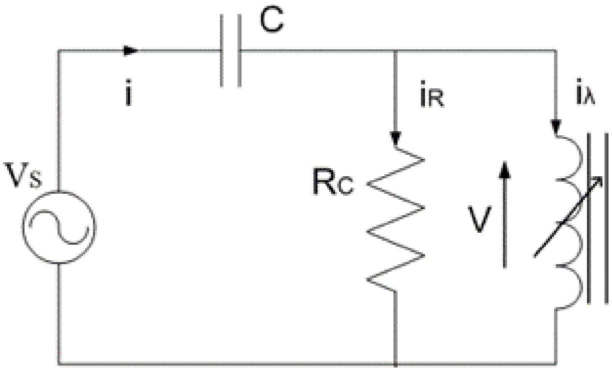


Fig. 3: Model Circuit for Ferroresonance.

In this paper, the magnetizing curve was obtained by recording voltage and current in the open circuit test of the transformer. To get the saturation, the open circuit voltage is increased up to 150% of the rated value. This is shown in Fig. 3 with a solid line.

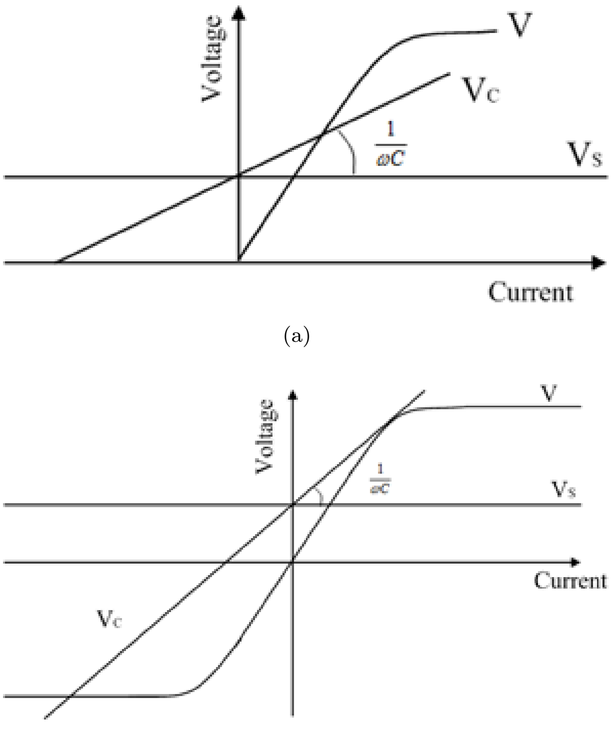
Now to fit this magnetic characteristic in the finite element simulator a suitable nonlinear equation is required. The help of the curve fitting tool of MATLAB has been taken. The tool suggested the following 7th-order polynomial relation.

$$i_\lambda = a\lambda + b\lambda^7, \tag{1}$$

Here λ = flux linkage, i_λ = magnetizing current, and a and b are constants. The approximated graph is also shown in Fig. 3 in a dotted line.

For developing the equivalent circuit of the transformer, the constant core loss model is used [18]. Here the nonlinearity of the transformer iron core is characterised as a nonlinear inductance. The core loss is represented as a constant resistance R_c connected in parallel to the non-linear inductance. The value of R_c is obtained from the no-load test of the transformer. The series resistance and leakage inductance of the transformer are neglected as it is assumed the transformer is operating at no load. The equivalent circuit is shown in Fig. 3.

The series capacitance value is decided from Rudenberg's graphical methods [5] as follows.



(a)

(b)

Fig. 4: The intersection of transformer core and capacitor characteristics (a) in the first quadrant and (b) in the third quadrant.

Applying Kirchhoff's voltage law (KVL) the voltage equation of the circuit shown in Fig. 1 can be obtained as

$$V_s = V_C + V, \tag{2}$$

Where V_C is the voltage drop across the series capacitor and V is the transformer voltage. The graphical solution of (2) can be obtained by plotting the magnetizing curve of the transformer, supply voltage, and capacitor characteristics on the same graph, as shown in Fig. 4(a). The operating point of the circuit will be the intersection of the magnetizing curve and capacitor curve. If the supply voltage is increased, the capacitor line along with the operating point will slide upward along the magnetizing curve. At one point there will no operating point be available in the first quadrant. Then extended capacitor line will intersect the magnetizing curve in the third quadrant as shown in Fig. 4(b). The operating point in the third quadrant produces large voltages in the circuit and this phenomenon is called ferroresonance. Thus the limiting condition will be achieved when the capacitor line is tangent to the magnetizing curve in the first quadrant.

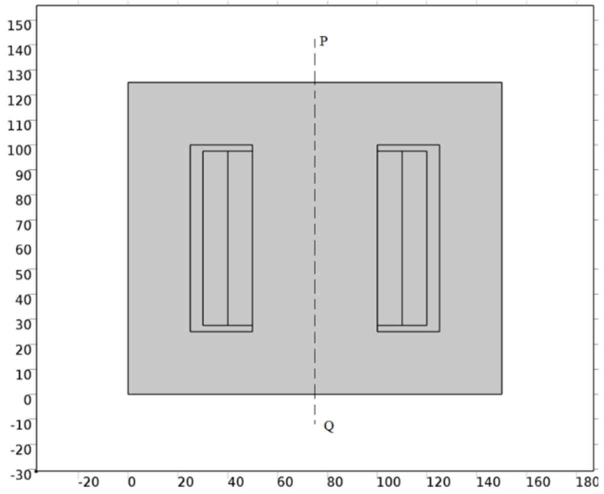


Fig. 5: Transformer for finite element analysis (units in mm).

Mathematically, (2) can be further processed and the following 2nd-order differential equation can be obtained.

$$\frac{d^2\lambda}{dt^2} = \frac{dV_s}{dt} - \frac{1}{C}\left(\frac{v}{R_C}\right) - \frac{1}{C}\left(a\lambda + b\lambda^7\right), \tag{3}$$

Solution of (3) can throw some light on the behavior of the system under ferroresonance. A couple of MATLAB simulation-based studies have been published by the authors [19, 20]. But those analyses with the nonlinear mathematics, could not reveal the distribution of flux in the transformer core under ferroresonance. For that finite element study has been attempted in the next sections.

3. Setup the System for Finite Element Analysis

The following setup has been created in the simulation environment.

Rated power = 200 VA

Frequency = 50 Hz

Primary voltage = 220 V

Secondary voltage = 24 V

Core material = silicon steel GO 3%

Primary turn = 416 with wire diameter 1.02 mm

Secondary turn = 45 with wire diameter 2.71 mm

For simplicity, 2D analysis has been carried out. For that, the transformer with a specific dimension as shown in Fig. 6 has been created in Comsol Multiphysics software. Equation (1) is incorporated into the core characteristic. To build up the ferroresonance circuit shown in Fig. 3, the Electric Circuit interface is coupled with the Magnetic Fields interface. As the flux

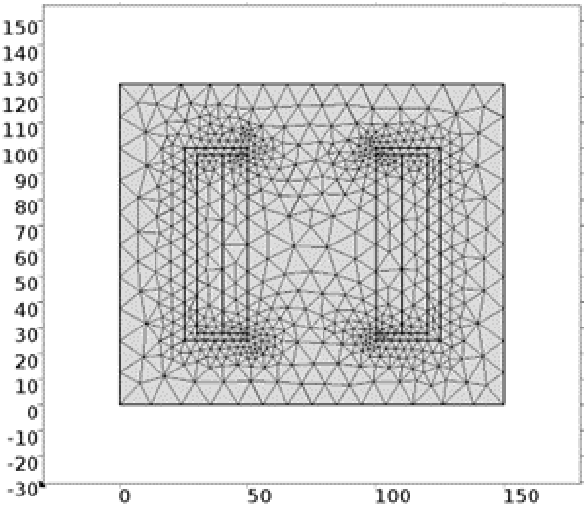


Fig. 6: Mesh generation for finite element analysis (units in mm).

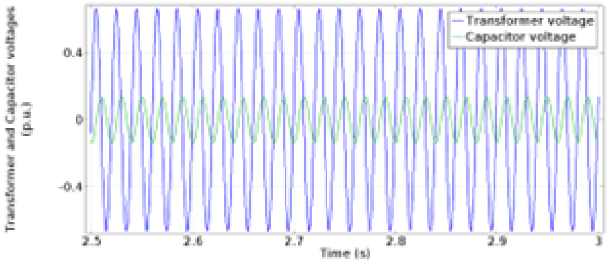


Fig. 7: Voltage plot at 0.6695 p.u. supply voltage - no ferroresonance.

density vector B will only have the tangential component along the axis of symmetry PQ (Fig. 5), Dirichlet's boundary condition is assigned along this line.

In the next step triangular mesh has been generated as shown in Fig. 6. To make a balance between the computational time and accuracy of the result element size is taken as 'fine' with 1916 domain elements and 274 boundary elements. The system is simulated for 998 degrees of freedom.

Assuming quasi-static analysis the displacement current density can be considered as zero. So Ampere's law can be rewritten as:

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \mathbf{J_e}, \tag{4}$$

Here σ is the electric conductivity and $\mathbf{J_e}$ is an externally generated current density.

Using the definitions of vector potential $\mathbf{A}$ and scalar potential V,

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{5}$$

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{dt}. \tag{6}$$

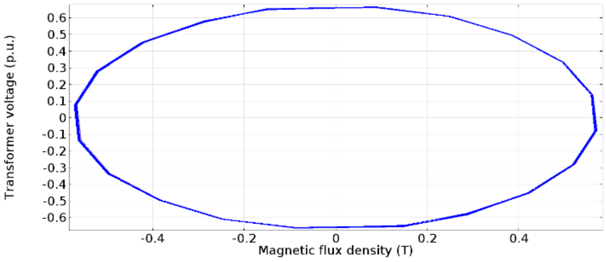


Fig. 8: Phase plane plot at 0.6695 p.u. supply voltage - no ferroresonance.

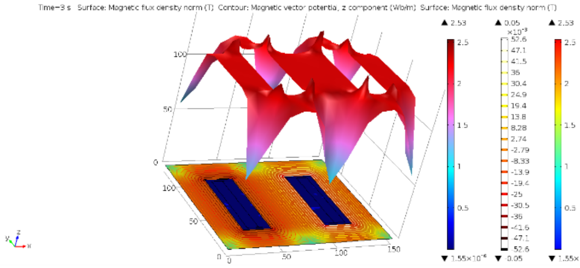


Fig. 9: Magnetic flux density at 0.6695 p.u. supply voltage - no ferroresonance.

Equation (4) can be rewritten as

$$\sigma \frac{\partial \mathbf{A}}{dt} + (\nabla \times \mathbf{H}) + \sigma \nabla V = \mathbf{J_e}. \tag{7}$$

The no-load test of the transformer is created in the simulation. The primary winding current is taken as the excitation parameter and the voltages across two windings are computed. The components of magnetic flux density are derived from the vector potential.

The stored magnetic energy density for the whole transformer can be calculated as [21]

$$W_m = L_{Fe} \int\int \frac{1}{2} \mu H^2, \tag{8}$$

Where L_{Fe} is the z dimension of the core of the transformer.

The main flux is obtained by integrating the normal component of the flux density as

$$\Phi_0 = L_{FE} \int \vec{B}.\widehat{n} dl. \tag{9}$$

The primary winding flux linkage

$$\lambda_0 = N_1 \Phi_0. \tag{10}$$

The observations are listed in the next section.

4. Results and Analysis

The supply voltage is increased gradually and the transformer and capacitor voltages are observed. Up to

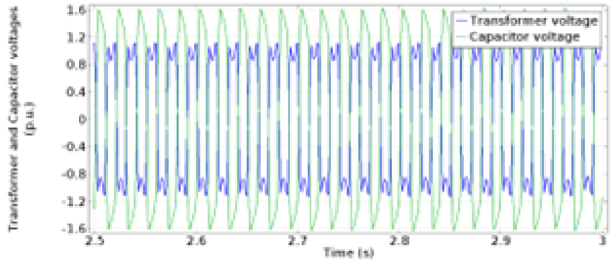


Fig. 10: Voltage plot at 0.67 p.u. supply voltage - with ferroresonance.

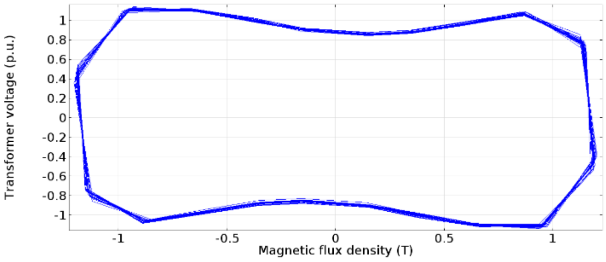


Fig. 11: Phase plane plot at 0.67 p.u. supply voltage - with ferroresonance.

0.6695 per unit (p.u.) supply voltage system shows no ferroresonance. The circuit behaves like an inductive circuit as the transformer voltage is greater than the capacitor voltage (Fig. 7). It can be concluded that the system is working in the 1st quadrant in the V-I characteristics shown in Fig 4. Taking transformer voltage and flux linkage as the system variables the phase trajectory can be obtained as shown in Fig. 8. In nonlinear mathematics the phase trajectory is a powerful tool for analyzing the stability of the system. The closed trajectory (Fig. 8) established the existence of a limit cycle [22]. Fig. 9 shows the distribution of magnetic flux density at all points of the core surface. The peak value of magnetic flux density after a steady state is 1.18 T at no-ferroresonance conditions.

At 0.67 p.u. supply voltage, ferroresonance is observed. It can be seen from Fig. 10 that the capacitor voltage peak value extended by almost 1.6 p.u. whereas transformer voltage reached 1.1 p.u peak value. The circuit becomes capacitive. It was expected as per Rudenberg's graphical analysis [5]. Here the system is working in the 3rd quadrant of the V-I characteristics shown in Fig. 4. The phase plane plot is shown in Fig. 11. The plot is no longer a circular one. This observation matches with different articles published earlier [23, 24]. Fig. 12 shows the magnetic flux density of the core surface under ferroresonance. The peak value of flux density achieved after the initial transient decay down is 2.47 T. This is well above the saturation value of GO steels which is 1.7 T [25]. The flux density peak obtained in ferroresonance is 2.1 times the no ferroresonance condition. Fig. 13 shows the change of flux density peak while the system enters into the ferroresonance condition from the non-ferroresonance condition.

5. Conclusion

In this paper, the non-linearity of the transformer is represented by a 7th-order polynomial equation which was obtained from the laboratory setup. The non-linear transformer model is then connected with a sinusoidal supply voltage and a series capacitor. The analytical model is tested with the finite element method and solved for magnetic vector potential.

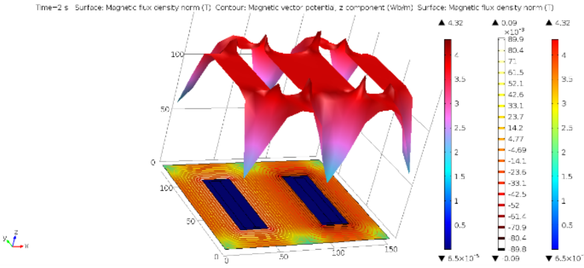


Fig. 12: Magnetic flux density at 0.67 p.u. supply voltage - with ferroresonance.

While varying the source voltage, keeping other parameters constant a sharp jump has been observed in transformer and capacitor voltages. Here the circuit's operating point changes from the first quadrant to the third quadrant as described in section 2. . The characteristic matches with the fundamental ferroresonance. So 0.67 p.u. supply voltage is the critical value for the proposed transformer for ferroresonance to occur along with other circuit parameters as mentioned. A detailed study of magnetic flux density shows that during ferroresonance the flux density raised to 2.1 times. This may throw the core of the transformer deep into the saturation. This will permanently magnetize the core

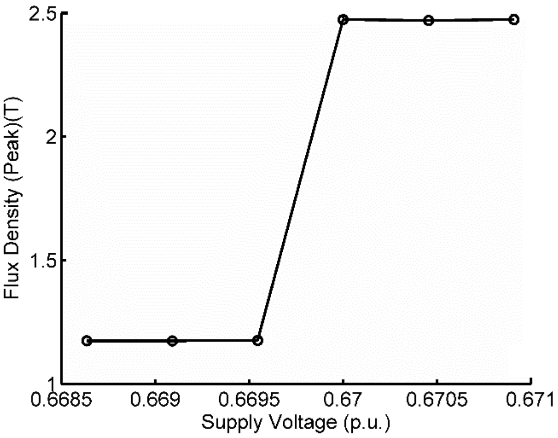


Fig. 13: Change of magnetic flux density with supply voltage.

material. Also increases the core losses. The jump in magnetic flux density also makes it difficult to rebuild the condition under a laboratory environment as it may permanently damage the core of the transformer under test.

Being a nonlinear phenomenon, the analysis of ferroresonance invites complex mathematical calculations. This paper attempted an easy method to investigate the flux distribution of the transformer core before and after ferroresonance. This can be useful to identify the ferroresonance possibilities and avoid the hazardous areas.

Author Contributions

R. S. Pal developed the theoretical formalism, performed the analytic calculations, and performed the numerical simulations. Both R. S. Pal and M. Roy contributed to the final version of the manuscript. M. Roy supervised the project.

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