

ROBUST TRACKING CONTROL FOR A PERMANENT MAGNET SYNCHRONOUS GENERATOR

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Abstract. This paper presents a study of a robust control strategy based on the Attractive Ellipsoid Method (AEM) to achieve trajectory tracking for a Permanent Magnet Synchronous Generator (PMSG). For the PMSG system, trajectory tracking control is necessary such that the maximum power point tracking can be carried out, which depends on controlling time-varying references (the generator angular speed), mainly when the generator motion is originated from a variable and intermittent energy source such as the wind. The proposed controller is based on the selection of the best gain matrix by numerically employing the AEM with the bilinear matrix inequality technique application. The controller guarantees that the trajectory tracking error reaches an ellipsoid of small enough size, which is determined in an optimized way, such that the practical stability for the tracking error is ensured. The effectiveness of the proposed controller is verified via simulations for a PMSG coupled with a three-phase rectifier, where the variables to be controlled reach the desired time-varying references, additionally a comparison is made against a classical Proportional Integral (PI) control scheme and a state feedback LQR controller.

Keywords

attractive ellipsoid method, robust control, BMI, PMSG, tracking.

1. Introduction

The wind energy conversion systems (WECS) are complex, highly nonlinear systems that require control strategies to fulfill different tasks as the pitch angle control, maximum power extraction by means of Maximum Power Point Tracking (MPPT) algorithms, speed controllers for the electrical generator, among others. These control objectives involve the stabilization and tracking of some desired system trajectories as speed, current, voltage, etc. Particularly, tracking has a special role in the wind energy conversion process due to the need of extracting the maximum available power in the wind at all time. The most common parameters used to track the maximum power point in WECS are the torque, speed and current [1, 2, 3].

In the recent years, the WECS have integrated the PMSG almost at the same rate than the Doubly Fed Induction Generator (DFIG). The main advantages of PMSG configuration are its gearless construction, high reliability and efficiency, and the elimination of the DC excitation system because the field is provided by the permanent magnets. Among the research developed for PMSG-based systems one can find works related to classical control schemes based on linear control theory such as the vector control, which is almost entirely based on PI controllers [4, 5].

Owing to the presence of disturbances, parameters variations, load changes and nonlinear models in WECS, robust and adaptive control strategies have also been applied to improve the system performance [6, 7, 8]. Sliding modes control is one of these robust strategies used to control PMSG-based WECS,

obtaining good results for the desired control objectives [9, 10].

Among the robust control techniques one can find the controller design based on the Attractive Ellipsoid Method (AEM) which is a tool mainly used in control theory for the design of robust feedback controllers with respect to a wide class of uncertainties in the model description. When uncertainties are present in a model, the system motion stabilization to zero is not always possible, and only boundedness of trajectories within some compact set is guaranteed. This boundedness is frequently provided by invariant sets. One way to obtain this characteristic is the invariant ellipsoid method which is named attractive ellipsoid method if this condition is guaranteed for all initial conditions [11, 12]. The AEM entails the representation of the control problems in terms of Linear or Bilinear Matrix Inequalities (BMI) which can be solved with semi-definite programming or BMI specialized solvers. Most applications of AEM address the stabilization problem [12, 13] and there are few works related to the trajectory tracking. In [14] the AEM for trajectory tracking in discrete-time stochastic systems is addressed.

This paper proposes the design of a robust controller based on the AEM to achieve trajectory tracking for nonlinear systems. The ellipsoid size is determined in an optimized way such that the tracking error remains inside a small region through of the synthesized controller, fulfilling a practical stabilization of the tracking error. Simulation results for a PMSG-rectifier system are presented to illustrate the robustness of the developed control methodology. Additionally, the performance of the proposed scheme is compared against a classical PI-based control scheme and a state feedback LQR controller.

2. PMSG system modeling

This section presents the system under study for which the AEM will be developed. The whole system to be modeled is composed by the PMSG, a three-phase rectifier and the control system, as depicted in Fig. 1.

A common representation of the PMSG dynamics is through the Park model also known as dq reference frame model. The PMSG stator voltage equations are expressed in the synchronous dq coordinates as [15]

$$\begin{cases} V_{ds} = R_s i_{ds} - \omega_r L_{qs} i_{qs} + L_{ds} \frac{di_{ds}}{dt} \\ V_{qs} = \omega_r L_{ds} i_{ds} + R_s i_{qs} + L_{qs} \frac{di_{qs}}{dt} + \omega_r \lambda_r \end{cases} \quad (1)$$

where $V_{ds}, V_{qs}, i_{ds}, i_{qs}$ are the d and q axes stator voltages and currents respectively; L_d and L_q are d and q axes inductance, R_s is the stator resistance, λ_r

is the magnetic flux and ω_r is the electrical angular speed.

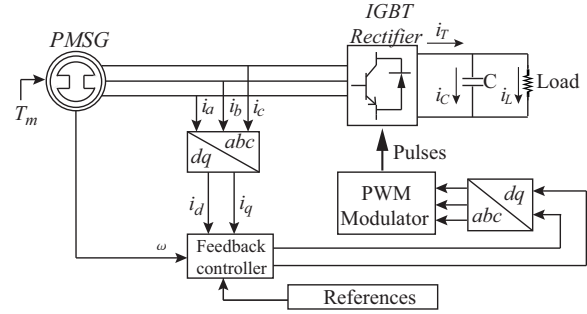


Fig. 1: Block diagram of a PMSG-based generation system

. From model (1), the stator currents can be expressed as

$$\frac{d}{dt} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_{qs}} & \frac{\omega_r L_{qs}}{L_{qs}} \\ -\frac{\omega_r L_{ds}}{L_{qs}} & -\frac{R_s}{L_{qs}} \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{ds}} & 0 \\ 0 & \frac{1}{L_{qs}} \end{bmatrix} \begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{\omega_r \lambda_r}{L_{qs}} \end{bmatrix}. \quad (2)$$

The electromagnetic torque for a generator with surface mounted permanent magnets, where the d and q axes inductances are equal, can be expressed as

$$T_e = \frac{3}{2} p \lambda_r i_{qs}. \quad (3)$$

The mechanical angular speed dynamics of the PMSG are given as

$$J \frac{d}{dt} \omega_m + F \omega_m = T_e - T_m \quad (4)$$

where J is the shaft moment of inertia, ω_m is the rotor angular speed, F is the viscous friction coefficient, T_e and T_m are the electromagnetic and mechanical torque, respectively. The mechanical rotor angular speed ω_m is related to the electrical angular speed (ω_r) and the number of poles (p) of the PMSG as $\omega_m p = \omega_r$.

Finally, the complete system model becomes

$$\frac{d}{dt} \begin{bmatrix} \omega_r \\ i_{ds} \\ i_{qs} \end{bmatrix} = \begin{bmatrix} -\frac{F\omega_r}{J} + \frac{3\lambda_r p i_{qs}}{2J} \\ -\frac{R_s i_{ds}}{L_{ds}} + \frac{\omega_r i_{qs} L_{qs}}{L_{ds}} \\ -\frac{\lambda_r \omega_r}{L_{qs}} - \frac{\omega_r L_{ds} i_{ds}}{L_{qs}} - \frac{R_s i_{qs}}{L_{qs}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{L_{ds}} & 0 \\ 0 & \frac{1}{L_{qs}} \end{bmatrix} \begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} + \begin{bmatrix} \frac{p T_m}{J} \\ 0 \\ 0 \end{bmatrix}. \quad (5)$$

The system model (5) is a nonlinear model due to the interaction among system states, therefore classical linear techniques as PI controllers will have a lower performance. Hence, the use of a modern control technique as the AEM is proposed.

3. Attractive Ellipsoid Method

This paper deals with the design of a robust feedback control scheme for trajectory tracking in nonlinear systems based on the, so-called, Attractive Ellipsoid Method which provides convergence of system trajectories from any initial condition to a positively invariant set, named ellipsoidal set, for a class of nonlinear models even in the case of incomplete system information or in the presence of external disturbances. This ellipsoidal set possesses minimal properties related to the ellipsoid size, volume, etc. These properties may be used in the design of feedback control strategies for a variety of applications, as shown in [16, 17].

3.1. Mathematical Preliminaries

This subsection presents basic definitions and important results needed for the synthesis of the AEM.

Let us consider a general nonlinear system described by

$$\begin{aligned} \dot{x} &= f(x, u) \\ x(0) &= x_0 \in \mathbb{R}^n \end{aligned} \quad (6)$$

where $g : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ is a suitable right hand side.

The class of systems which are suitable for the proposed feedback controller design are limited to a specific class of nonlinear systems named quasi-Lipschitz dynamic models with bounded uncertainties. The formal description of the quasi-Lipschitz function is presented as follows [18].

Definition 1. *Quasi-Lipschitz function* A vector function $f : \mathbb{R}^n \rightarrow \mathbb{R}^k$ is said to be a quasi-Lipschitz function $\mathcal{C}(A, c_0, c_1)$ if there exists a matrix $A \in \mathbb{R}^{k \times n}$ and nonnegative constants c_0 and c_1 , such that for every $x \in \mathbb{R}^n$, the following inequality holds:

$$\|f(x) - Ax\|^2 \leq c_0 + c_1 \|x\|^2. \quad (7)$$

Taking into account Definition 2 for systems containing nonlinear control actions u , the quasi-Lipschitz condition can be expanded to incorporate bounds related to such control inputs. Then, a linear part of the input (B) can be added to quasi-Lipschitz condition together with a new variable to bound the corresponding nonlinear part (c_2) resulting in the following form

$$\|f(x, u) - Ax - Bu\|^2 \leq c_0 + c_1 \|x\|^2 + c_2 \|u\|^2 \quad (8)$$

The constant matrices A and B characterize the “nominal linear part of a system”, while the nonnegative constants c_k ($k = 0, 1, 2$) define the permitted deviation of any nonlinearity with respect to the nominal linear system, which are supposed to be known a priori. Since function $f \in \mathcal{C}(A, B, c_0, c_1, c_2)$, it means that the growth rates of these function is not faster than linear. To guarantee the control design, the pair (A, B) must be determined such that the system is controllable.

3.2. Attractive Ellipsoid for trajectory tracking

For different applications, the trajectory tracking of the system variables to a desired reference (set point) is necessary. To this end, let us define the tracking error as

$$e = x_r - x \quad (9)$$

where x_r is the desired reference for x . Then, it is necessary to develop a feedback control strategy to fulfill such objective. This paper proposes the synthesis of a robust controller defined as

$$u = K e \quad (10)$$

where $K \in \mathbb{R}^{m \times n}$ is the controller gain matrix, which must guarantees the boundedness of the error trajectories of the closed-loop system (??)–(10), even in the presence of uncertainties satisfying the quasi-Lipschitz conditions.

Definition 2. *The motion of e in (9), for $t \geq 0$, belongs asymptotically to the attractive ellipsoid*

$$\mathcal{E}(P) = \{x \in \mathbb{R}^n : x^T P x \leq 1, P = P^T > 0\}$$

with center at 0, and the corresponding symmetrical matrix P , if the following inequality is satisfied

$$\limsup_{t \rightarrow \infty} e^T P e \leq 1.$$

If this ellipsoid exists for a given system, it may be seen as a generalization of the uniformly ultimately boundedness (UUB) property, since once the system trajectories enters to the ellipsoid, they remain inside but do not converge to a specific point [19].

4. Feedback controller design based on the AEM

This section focuses on the problem of designing the robust controller based on the AEM, with the structure shown in (10), to achieve trajectory tracking for the PMSG system (5).

The PMSG model presented in (5) may be rewritten to obtain the linear nominal part and the nonlinear part of the system as

$$f(x, u) = Ax + Bu + \Delta f_1 + \zeta \quad (11)$$

with

$$A = \begin{bmatrix} -\frac{F}{J} & 0 & -\frac{3p\lambda}{2J} \\ 0 & -\frac{R_s}{L_{ds}} & 0 \\ \frac{\lambda_r}{L_{qs}} & 0 & -\frac{R_s}{L_{qs}} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ \frac{1}{L_{ds}} & 0 \\ 0 & \frac{1}{L_{qs}} \end{bmatrix}$$

$$\Delta f_1 = \begin{bmatrix} 0 \\ \frac{\omega_r L_{qs} i_{qs}}{L_{ds}} \\ -\frac{\omega_r L_{ds} i_{ds}}{L_{qs}} \\ 0 \end{bmatrix}.$$

where $x = [\omega_r \ i_d \ i_q]^T$ and $u = [V_{ds} \ V_{qs}]^T$. Subsequently, the term corresponding to the system disturbances represented by the vector ζ is added to the nonlinearities vector Δf_1 as

$$\Delta f = \Delta f_1 + \zeta = \begin{bmatrix} 0 \\ \frac{\omega_r L_{qs} i_{qs}}{L_{ds}} \\ -\frac{\omega_r L_{ds} i_{ds}}{L_{qs}} \end{bmatrix} + \begin{bmatrix} \frac{pT_m}{J} \\ 0 \\ 0 \end{bmatrix}. \quad (12)$$

The new vector Δf has to fulfill the quasi-Lipschitz condition, hence is necessary to know some information about the PMSG bounds. In the particular case of the PMSG, the system states (currents, voltages, speed) are physically limited or bounded by the machine capacity by means of crowbar systems, braking choppers, speed regulators among others in order to provide a safe operating condition. Based on this, the task of obtaining system bounds can be developed based on the machine's capacity and parameters.

Thus, by introducing vector Δf , system (5) can be presented as in (??), where the vectors associated to the nonlinearities of the system and the disturbances are assumed to satisfy the quasi-Lipschitz condition, as established in (7). Once system (5) is presented in the quasi-Lipschitz form (??), the robust control strategy can be designed.

The following lemma, considered as one of the main contributions of this paper, establishes the conditions

for determining the controller gain K and the attractive ellipsoid.

Lemma 1. *If for a positive definite matrix $P \in \mathbb{R}^{n \times n}$, a gain $K \in \mathbb{R}^{m \times n}$ and the nonnegative constants $\alpha, \tau_1, c_0, c_1 \in \mathbb{R}$, the following matrix inequality holds*

$$\begin{bmatrix} \bar{W}_{11} & \bar{W}_{12} & P \\ \bar{W}_{21} & \bar{W}_{22} & -P \\ P & -P & -\tau_1 I \end{bmatrix} < 0 \quad (13)$$

with

$$\begin{aligned} \bar{W}_{11} &= (A - BK)^T P + P(A - BK) + \alpha P \\ &\quad + \tau_1 c_1 \\ \bar{W}_{12} &= -A^T P + K^T B^T P + PBK - \alpha P \\ \bar{W}_{21} &= -PA + PBK + K^T B^T P - \alpha P \\ \bar{W}_{22} &= -K^T B^T P - PBK + \alpha P \\ \tau_1 &> 0, \alpha > 0 \end{aligned}$$

then the storage function

$$V(e) := e^T P e$$

satisfies the following inequality

$$\begin{aligned} \dot{V}(e) &\leq -\alpha V(e) + \beta \\ \beta &= \tau_1 c_0 \end{aligned}$$

and $\mathcal{E}(P)$ becomes the invariant ellipsoid for the closed-loop system (??)-(10).

Proof. Consider a quadratic storage function defined as

$$V(e) = e^T P e, \quad P = P^T > 0. \quad (14)$$

By obtaining the time-derivative of (14), it results in

$$\begin{aligned} \dot{V}(e) &= x^T (A^T P + PA - PBK - K^T B^T P) x \\ &\quad + x^T (-A^T P + K^T B^T P + PBK) x_r \\ &\quad + x_r^T (-PA + K^T B^T P + PBK) x \\ &\quad + x_r^T (-K^T B^T P - PBK) x_r - \Delta f^T P x_r \\ &\quad + \Delta f^T P x - x_r^T P \Delta f + x^T P \Delta f. \end{aligned} \quad (15)$$

Introducing a new vector $z := [x, x_r, \Delta f]$, then (15) can be represented in matrix form as

$$\dot{V}(e) = \begin{bmatrix} x^T \\ x_r^T \\ \Delta f^T \end{bmatrix}^T \overbrace{\begin{bmatrix} W_{11} & W_{12} & P \\ W_{21} & W_{22} & -P \\ P & -P & 0 \end{bmatrix}}^W \begin{bmatrix} x \\ x_r \\ \Delta f \end{bmatrix} \quad (16)$$

$$W_{11} = A^T P + PA - K^T B^T P - PBK$$

$$W_{22} = -K^T B^T P - PBK$$

$$W_{12} = -A^T P + K^T B^T P + PBK$$

$$W_{21} = -PA + K^T B^T P + PBK$$

adding and subtracting the terms $\alpha V(e), \tau_1 \|\Delta f_1\|^2$ in the right-hand side of the last equation leads to

$$\dot{V}(e) = z^T \overbrace{\begin{bmatrix} W_{11} + \alpha P & W_{12} - \alpha P & P \\ W_{21} - \alpha P & W_{22} + \alpha P & -P \\ P & -P & -\tau_1 I \end{bmatrix}}^{W_1} z - \alpha V(e) + \tau_1 \|\Delta f\|^2 \quad (17)$$

where

$$\begin{aligned} -\alpha V(e) &= -\alpha e^T P e \\ &= -\alpha (x_r^T P x_r - x_r^T P x - x^T P x_r + x^T P x). \end{aligned} \quad (18)$$

Taking into account the quasi-Lipschitz condition related to the uncertain part, Δf , then (17) becomes

$$\dot{V}(e) \leq z^T W_1 z + \tau_1 [c_0 + c_1 \|x\|^2] - \alpha V(e). \quad (19)$$

Finally, by appropriately rearranging the above terms, it results in

$$\dot{V}(e) \leq z^T \overbrace{\begin{bmatrix} \bar{W}_{11} & \bar{W}_{12} & P \\ \bar{W}_{21} & \bar{W}_{22} & -P \\ P & -P & -\tau_1 I \end{bmatrix}}^{\bar{W}} z - \alpha V(e) + \tau_1 c_0 \quad (20)$$

$$\begin{aligned} \bar{W}_{11} &= W_{11} + \alpha P + \tau_1 c_1, \\ \bar{W}_{12} &= W_{12} - \alpha P, \quad \bar{W}_{21} = W_{21} - \alpha P, \\ \bar{W}_{22} &= W_{22} + \alpha P, \\ \tau_1 &> 0, \alpha > 0. \end{aligned}$$

By determining P and K such that inequality $\bar{W} < 0$ is satisfied, then

$$\dot{V}(e) \leq -\alpha V(e) + \beta, \quad \beta = \tau_1 c_0. \quad (21)$$

and $\mathcal{E}(P)$ is the ellipsoid for the closed-loop system (??)–(10), which guarantees the convergence and boundedness of the trajectory tracking error. \square

4.1. AEM Optimization Problem

The determination of P and K such that the requirement of being $\bar{W} < 0$ in inequality (20) can be solved as an optimization problem, which is related to the minimization of the ellipsoid size, or equivalently, the minimization of the trace of P under the constraints defined by τ_1, α in (20), formally stated as

$$\min_{P, K, \tau_1, \alpha} \text{trace}(P) \quad (22)$$

subject to $(\bar{W} < 0)$.

Note that (13) is a bilinear matrix inequality due to the variables multiplication (this is, αP), which increases the difficulty to solve the optimization problem.

The solver used in this paper to obtain the solution of the BMI system 22 is PenBMI from TOMLABTM. PenBMI is a tool for solving optimization problems with quadratic objectives and linear and bilinear matrix inequality constraints. The algorithm uses a combination of the exterior penalty and interior barrier methods with the augmented Lagrangian method, the interested reader can consult a detailed explanation of the algorithm in [20].

Remark 1. *It is important to note that the solution of (22) for obtaining P and K is performed offline and then the implementation of the controller is reduced to that of a linear feedback controller as shown in (10).*

5. Simulation Results

To verify the performance of the designed robust control strategy, a simulation scheme is developed in the MATLAB/Simulink environment. Additionally, the performance of the proposed controller is compared with respect to the classical vector control based on PI controllers and a state feedback Linear Quadratic Regulator (LQR) controller. The selected PI gains are $K_{p1} = 10$, $K_{I1} = 20$, $K_{p2} = 0.6$, $K_{I2} = 3$, $K_{p3} = 12.4$, $K_{I3} = 15$. The LQR controller gain matrix was computed in Matlab using the selected linear part of the PMSG model contained in (11) with weighting matrices $Q = \text{diag}(500, 500, 500)$ and $R = \text{diag}(1, 1)$, which results as

$$K_{LQR} = \begin{bmatrix} 0 & 14.2032 & 0 \\ 21.7476 & 0 & 18.9564 \end{bmatrix}.$$

The first simulation test is under a constant wind speed selected at 10.6 m/s which develops a constant mechanical input torque of 2.04 N.m . The PMSG-based system parameters are presented in Table 1. The initial values are given as $x_1[0] = 0, x_2[0] = 0, x_3[0] = 0$, while the desired reference values assigned to the variables of interest (speed and d-axis current) are $i_{d_{ref}} = 0 \text{ A}$ and $\omega_{ref} = 140 \text{ rad/s}$. The constants that define the quasi-Lipschitz representation of the uncertain parts of the system, that is (7), are selected taking into account the PMSG parameters, which result in $c_0 = 5.02 \times 10^3$, $c_1 = 1.0 \times 10^3$.

Once solving (22), the following values are obtained

$$K_{AEM} = \begin{bmatrix} 0.549 & -0.587 & -0.043 \\ 15.6512 & 5.7102 & -0.118 \end{bmatrix}.$$

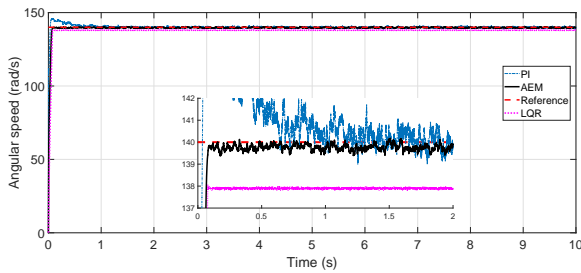
$$P = \begin{bmatrix} 0.5004 & 0.1150 & 0 \\ 0.1150 & 0.3605 & 0.0140 \\ 0 & 0.0140 & 0.1035 \end{bmatrix} \times 10^{-5}$$

$$\tau_1 = 6.0442 \times 10^{-6} \quad \alpha = 1.05 \times 10^{-3}.$$

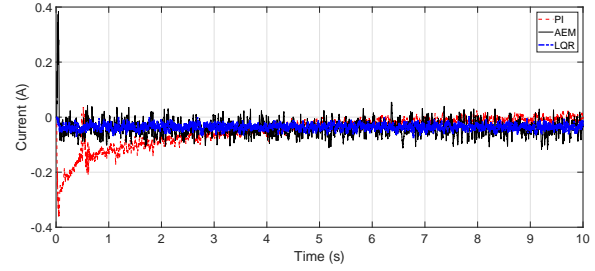
Tab. 1: System parameters

Parameter	Value
Nominal Power	1.1 <i>K.W</i>
Viscous friction	0.001147 N.m.s
Pole pairs	2
Magnetic flux	0.1852 <i>Wb</i>
Inertia	$1.854 \times 10^{-4} \frac{kg}{m^2}$
Stator resistance	1.6 Ω
<i>d</i> -axis inductance	0.006365 <i>H</i>
<i>q</i> -axis inductance	0.006365 <i>H</i>
Capacitance	1.1×10^{-3} <i>F</i>
Load resistance	120 Ω

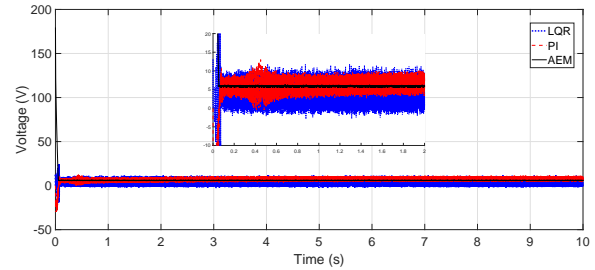
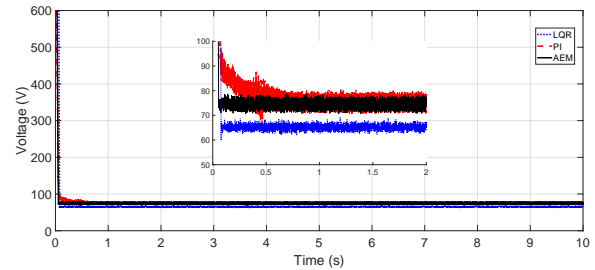
The trajectory tracking of state $x_1(t)$ corresponding to the rotational speed of the PMSG under the proposed feedback control strategy, the classical PI control scheme and the LQR controller is presented in Fig. 2. Owing to the fact that the AEM only guarantees a practical stability of the system, i.e., the boundedness of the trajectories to a region defined by an ellipsoid, and not the convergence to a specific point the proposed controller presents steady-state error. Despite of the steady-state error, considered here as a practical error stability, the proposed controller has better transient characteristics than the PI scheme, such as lower settling time and lower overshoot. On the other hand, the proposed controller has smaller tracking error than the LQR controller while the settling time is similar.

**Fig. 2:** PMSG rotational speed.

The trajectories of state $x_2(t)$, corresponding to the *d*-axis current of the PMSG, are presented in Fig. 3. Due to the integral term, the PI controller presents a smaller steady-state error than the proposed controller and LQR controller, but is slower to converge to the reference. The converter voltages developed by the three

**Fig. 3:** *d*-axis current.

controllers are shown in Fig. 4. It can be seen that the magnitude of the control actions are almost equal for the proposed AEM controller and PI control with a slight difference in their settling time, while the *q*-axis component of the LQR controller is approximately 10V down which is directly related with the greater speed tracking error.

(a) *d*-axis Voltage(b) *q*-axis Voltage**Fig. 4:** Converter voltages in dq-frame

In order to highlight the robustness of the designed controller, the input torque is changed from a constant value to a variable input on the range of $(-2.3, -1.1)$, with the form shown in the Fig. 5. This kind of input will be present in a real wind turbine system due to the inherent variable nature of wind speed.

The response of the rotational speed of the PMSG and the *d*-axis current are shown in Fig. 6a and Fig. 6b, respectively. Both figures show the comparison between the responses of the proposed feedback, the PI-based and the LQR controllers under the variable

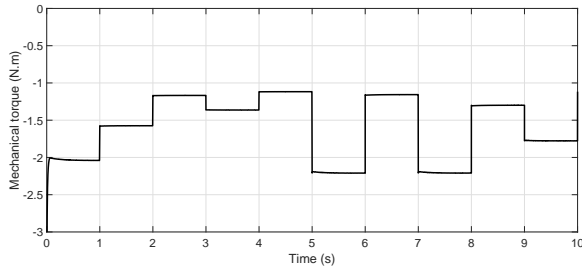
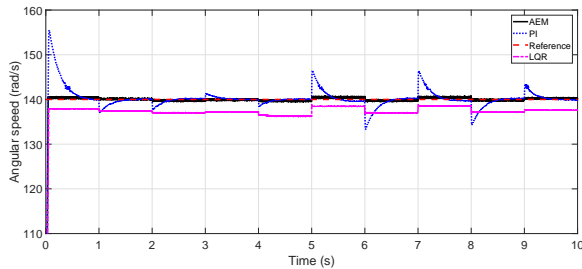
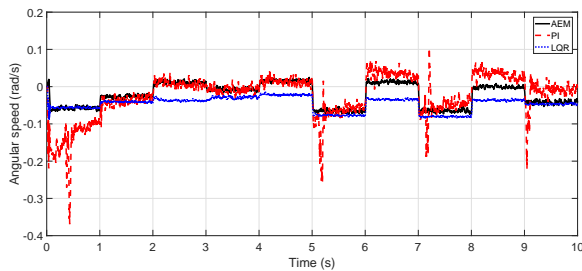


Fig. 5: Variable mechanical torque signal.

torque signal presented above. It can be seen that the performance of the PI-based scheme and the LQR controller becomes deteriorated due to the change in the operating point, while the proposed feedback controller maintains a good performance.



(a) PMSG rotational speed.



(b) d-axis current

Fig. 6: System responses for variable torque case.

Finally, Fig. 7 shows the proposed controller capability to track a time-varying speed reference with a fast response and small error which are better results than the PI and LQR controllers.

6. Conclusions

This paper proposes a robust state feedback controller based on the attractive ellipsoid method for a PMSG-rectifier system. The designed controller has simple structure based on a feedback law, nevertheless, possesses robustness which improves the performance of the system and has good tracking capabilities of the system variables toward the desired references. The

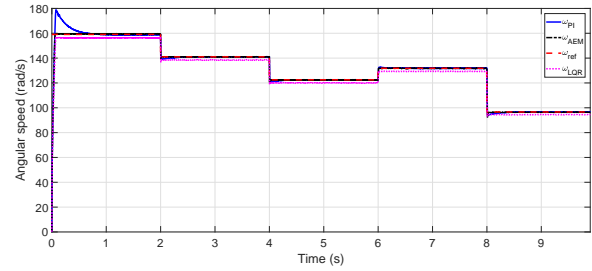


Fig. 7: PMSG speed tracking.

synthesized controller maintains an acceptable efficiency even under disturbances and has better performance than the classical PI-based control scheme and more accurate tracking than a feedback LQR controller. Simulation results in a PMSG-based wind energy conversion system illustrate the controller performance.

Author Contributions

D.C-V. contributed with the methodology, numerical simulations, validation and writing. H.A. contributed with conceptualization, methodology, formal analysis and writing. J.A-M. contributed with conceptualization, methodology, and writing.

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