SIMULATION OF MATERIAL INFLUENCES IN MR TOMOGRAPHY

K. Bartušek[a], E. Kadlecová[b], M. Steinbauer[b]

[a] Institute of Scientific Instruments, Academy of Sciences of the Czech Republic
Královopolská 147, 612 00 Brno, Czech Republic
e-mail: bar@isibrno.cz

[b] Brno University of Technology, Faculty of Electrical Engineering and Communication
Kolejní 2906/4, 612 00 Brno, Czech Republic
e-mail: kadleca@feec.vutbr.cz, steinbau@feec.vutbr.cz

Summary Materials with different magnetic susceptibility can cause deformation of magnetic field in MR tomograph, resulting in errors in obtained image. Using simulation and experimental verification we can solve the effect of changes in homogeneity of static magnetic fields caused by specimen made from magnetic material. This paper describes theoretical base of the magnetic resonance imaging method for susceptibility measurement. The method uses deformation of magnetic induction field in specimen vicinity. For MR purposes it is necessary to immerse specimen into reference medium with measurable MR signal.

1. INTRODUCTION

Proposed method of susceptibility measurement is based on presumption of constant magnetic flux in working space of superconducting magnet. Inserting of the specimen of thickness *a* and with magnetic susceptibility χ_{m1} causes local deformation of previously homogeneous magnetic field (idealized case is in figure 1).

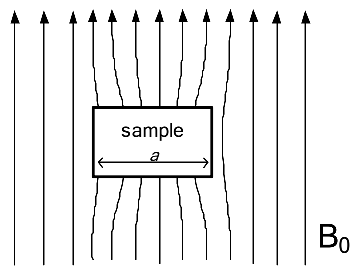

Fig 1. Magnetic flux density field deformation due to paramagnetic specimen

Primal magnetic field values are H_0 and B_0 and both have direction $\mathbf{u}_z$. In figure 2, magnetic intensity $\mathbf{H}_z(x)$ as well as magnetic flux density $\mathbf{B}_z(x)$ shapes are shown. Difference of this shapes from H_0 (B_0 respectively) values we call *reaction field*.

As we can see, the specimen affect not only field in his volume, but in his vicinity too. We assume that y size of the specimen is sufficient so we can neglect boundary effect and use 2D solution.

Magnetic flux density inside the specimen (neglecting little curvature of the shape in maximum) will be

$$\mathbf{B}_S = \mathbf{B}_0 \left(1 + \chi_{m1}\right). \tag{1}$$

Assume constant magnetic flux Φ thru normal area of cross-section S of the magnet working space

$$\Phi = \iint_{S_z} \mathbf{B} \cdot \mathrm{d}\mathbf{S} = konst. \tag{2}$$

It is evident that magnetic flux density outside the specimen is lowered resulting in shape, which can be introduced as superposition of homogeneous field $\mathbf{B}_0$ and deformation field $\Delta\mathbf{B}$.

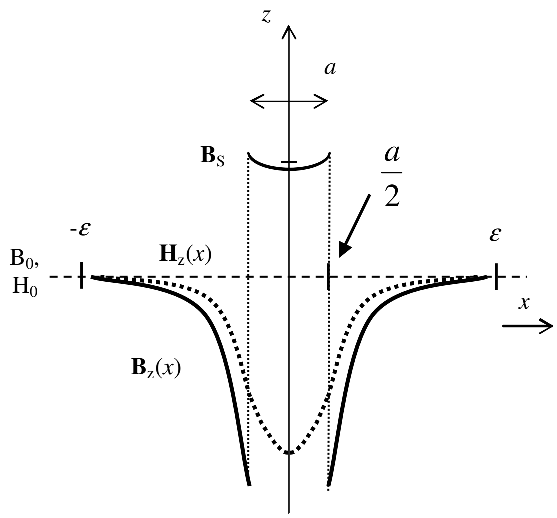

Fig 2. Ideal shape of magnetic intensity and flux density in paramagnetic specimen and its vicinity

If we cannot measure value $\mathbf{B}_S$ directly, it is impossible to determine susceptibility taking advantage from (1). So it is necessary to use indirect measure method. For z-x cross-section figure 2 in the middle of the specimen we can write

$$\int_{-\varepsilon}^{\varepsilon} \Delta\mathbf{B}(x)\,\mathrm{d}x \cong 0\,, \tag{3}$$

what means that sum of areas bounded by curve in this figure with respect to the base value of induction B_0 is zero, where ε is sufficient distance from specimen with respect to its impact on induction change.

Knowing the course of $\Delta\mathbf{B}(x)$ (we can obtain it using suitable MRI technique and reference substance giving MR signal in vicinity of material), we can also enumerate χ_{m1} value of the investigated specimen material

$$\chi_{\mathrm{m1}} \cong -\frac{\int_{-\varepsilon}^{-a/2} \Delta B_z \,\mathrm{d}x + \int_{a/2}^{\varepsilon} \Delta B_z \,\mathrm{d}x}{a\,\mathbf{B}_0}. \qquad (4)$$

2. ANALYTIC MODEL

Let's have specimen with susceptibility χ_{m1} surrounded by medium with susceptibility χ_{m2} and placed into static primal magnetic field with magnetic intensity vector $\mathbf{H}_0$ oriented in $\mathbf{u}_z$ direction. We have to determine magnetic intensity $\mathbf{H}$ of incurred field, which is superposition of primal and reaction field $\mathbf{H}_r$ (effect of specimen magnetization).

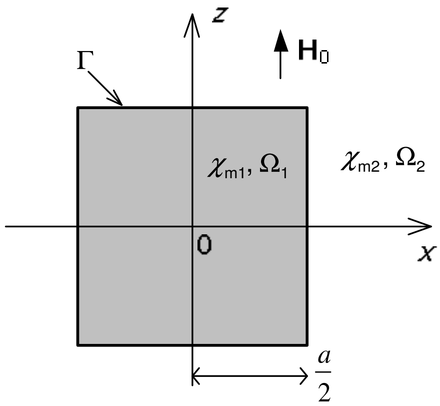

Fig. 3 2D analytic model

Because there are not variable currents in whole area, magnetic field is irrotational (rot $\mathbf{H} = 0$) and we can use scalar magnetic potential [5]

$$\mathbf{H} = -\mathrm{grad}\,\varphi_{\mathrm{m}}. \qquad (5)$$

Magnetic potential of primal field of intensity $\mathbf{H}_0$ is using (5)

$$\varphi_{\mathrm{m0}} = -\int \mathbf{H}_0 \cdot \mathbf{u}_z \,\mathrm{d}z = -H_0\, z. \qquad (6)$$

Incidence of magnetized specimen from figure 3 we can replace with effect of field of surface magnetic charge with density σ_{m} on boundary of areas Ω_1 a Ω_2 - see figure 4, whereas susceptibility of areas is now zero.

First we have to compute magnetic charge density distribution on bound Γ and consequently the intensity of reaction field $\Delta\mathbf{H} = \mathbf{H} - \mathbf{H}_0$

$$\Delta\mathbf{H}(\mathbf{r}) = \frac{1}{2\pi} \oint_{\Gamma} \sigma_{\mathrm{m}}(\mathbf{r}') \frac{\mathbf{u}_{\mathrm{r}}}{R(\mathbf{r},\mathbf{r}')} \mathrm{d}\Gamma. \qquad (7)$$

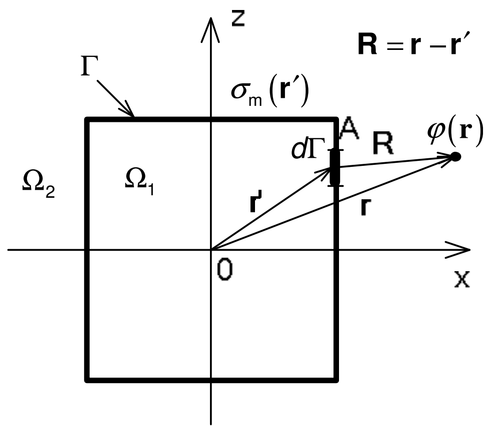

Fig. 4 Replacement of magnetization effect by surface magnetic charge at the area boundaries

Surface magnetic charge invokes scalar magnetic potential

$$\varphi_{\mathrm{mr}}(\mathbf{r}) = -\frac{1}{2\pi} \oint_{\Gamma} \sigma_{\mathrm{m}}(\mathbf{r}') \ln R(\mathbf{r},\mathbf{r}') \mathrm{d}\Gamma. \qquad (8)$$

Total scalar magnetic potential at point $\mathbf{r}$ is superposition of static primal field intensity (6) and contribution from charged bound (8)

$$\varphi_{\mathrm{m}}(\mathbf{r}) = -H\, z - \frac{1}{2\pi} \oint_{\Gamma} \sigma_{\mathrm{m}}(\mathbf{r}') \ln R(\mathbf{r},\mathbf{r}') \mathrm{d}\Gamma. \qquad (9)$$

Integral formula for surface magnetic charge density we obtain using condition of magnetic flux $\mathbf{B}_{\mathrm{n}} = B_{\mathrm{n}}\mathbf{u}_{\mathrm{n}}$ normal component conjunction on bound Γ (see figure 5)

$$B_{\mathrm{n}} = \mu_0 (1 + \chi_{\mathrm{m1}}) H_{\mathrm{1n}} = \mu_0 (1 + \chi_{\mathrm{m2}}) H_{\mathrm{2n}}. \qquad (10)$$

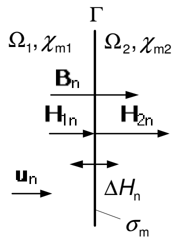

Fig. 5 Normal components of boundary magnetic intensity and flux

By analogy to the Gauss theorem causes magnetic charge of density σ_{m} at point A magnetic field of intensity

$$\Delta H_{\mathrm{n}} = \pm \frac{\sigma_{\mathrm{m}}(\mathrm{A})}{2}. \qquad (11)$$

Using (9) and (5) we can derive the normal components of magnetic field intensity in both areas at point A (see figure 5)

$$H_{1\mathrm{n}} = H_0\,\mathbf{u}_\mathrm{z}\cdot\mathbf{u}_\mathrm{n} + \\ +\frac{1}{2\pi}\,\mathrm{grad}\oint_{\Gamma,\mathbf{r}\in\Omega_1}\sigma_\mathrm{m}(\mathbf{r}')\ln R(\mathbf{r},\mathbf{r}')\,\mathrm{d}\Gamma\,\mathbf{u}_\mathrm{n}, \quad (12)$$

$$H_{2\mathrm{n}} = H_0\,\mathbf{u}_\mathrm{z}\cdot\mathbf{u}_\mathrm{n} + \\ +\frac{1}{2\pi}\,\mathrm{grad}\oint_{\Gamma,\mathbf{r}\in\Omega_2}\sigma_\mathrm{m}(\mathbf{r}')\ln R(\mathbf{r},\mathbf{r}')\,\mathrm{d}\Gamma\,\mathbf{u}_\mathrm{n}. \quad (13)$$

Whenever A ∈ Γ and thus **r** ∈ Γ, has integral in formulas (12) a (13) singularity at point A (where **r** = **r**'). We can remove this singularity omitting point **r** = **r**' from integration and taking field contribution of this point using (11) instead. So we can write

$$H_{1\mathrm{n}} = H_0\,\mathbf{u}_\mathrm{z}\cdot\mathbf{u}_\mathrm{n} + \\ +\frac{1}{2\pi}\oint_{\substack{\Gamma\\ \mathbf{r}\neq\mathbf{r}'}}\sigma_\mathrm{m}(\mathbf{r}')\frac{1}{R(\mathbf{r},\mathbf{r}')}\mathrm{d}\Gamma\,\mathbf{u}_R\cdot\mathbf{u}_\mathrm{n} - \frac{\sigma_\mathrm{m}(\mathrm{A})}{2}, \quad (14)$$

$$H_{2\mathrm{n}} = H_0\,\mathbf{u}_\mathrm{z}\cdot\mathbf{u}_\mathrm{n} + \\ +\frac{1}{2\pi}\oint_{\substack{\Gamma\\ \mathbf{r}\neq\mathbf{r}'}}\sigma_\mathrm{m}(\mathbf{r}')\frac{1}{R(\mathbf{r},\mathbf{r}')}\mathrm{d}\Gamma\,\mathbf{u}_R\cdot\mathbf{u}_\mathrm{n} + \frac{\sigma_\mathrm{m}(\mathrm{A})}{2}, \quad (15)$$

where was used

$$\mathrm{grad}\,\ln R(\mathbf{r},\mathbf{r}') = \frac{1}{R(\mathbf{r},\mathbf{r}')}\mathbf{u}_R. \quad (16)$$

Substituting from (14) and (15) into (10) we have after some rearrangement

$$\frac{\chi_\Delta}{2\pi}\oint_{\substack{\Gamma\\ \mathbf{r}\in\Gamma,\mathbf{r}\neq\mathbf{r}'}}\frac{\sigma_\mathrm{m}(\mathbf{r}')}{R(\mathbf{r},\mathbf{r}')}\mathrm{d}\Gamma\,\mathbf{u}_R\cdot\mathbf{u}_\mathrm{n} + \frac{\sigma_\mathrm{m}(\mathbf{r})}{2} = \\ = -\chi_\Delta H_0\,\mathbf{u}_\mathrm{z}\cdot\mathbf{u}_\mathrm{n}, \quad (17)$$

where differential susceptibility was introduced

$$\chi_\Delta = \frac{\chi_{\mathrm{m}1}-\chi_{\mathrm{m}2}}{\chi_{\mathrm{m}1}+\chi_{\mathrm{m}2}+2}. \quad (18)$$

Formula (17) is not analytically solvable, thus we solve it numerically by mean of boundary element method (BEM). Let's divide boundary Γ to N segments of the same length Δ_l and with constant surface charge density $\sigma_\mathrm{m}(i)$ on segment i.

Discretizing (17) we have for this segment

$$\frac{\sigma_\mathrm{m}(i)}{2} + \frac{\chi_\Delta}{2\pi}\sum_{j=1,j\neq i}^{N}\frac{\sigma_\mathrm{m}(j)\Delta_l}{R(\mathbf{r}_i,\mathbf{r}_j)}\mathbf{u}_R\cdot\mathbf{u}_{\mathrm{n}i} = \\ = -\chi_\Delta H_0\,\mathbf{u}_\mathrm{z}\cdot\mathbf{u}_{\mathrm{n}i} \quad (19)$$

Now we can write matrix formulae

$$\mathbf{K}\,\mathbf{q} = \mathbf{h} \quad (20)$$

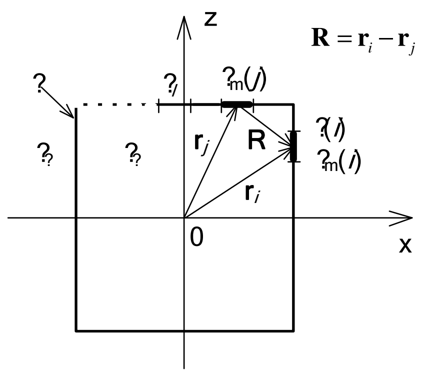

Fig. 6 Boundary element method

K is square matrix with components

$$k_{ij} = \frac{1}{2}\delta(i,j) + \frac{\chi_\Delta\Delta_l\,\mathbf{u}_R\cdot\mathbf{u}_{\mathrm{n}i}}{2\pi R(\mathbf{r}_i,\mathbf{r}_j)}\left(1-\delta(i,j)\right) \quad (21)$$

$\delta(i,j)$ is Kronecker's operator

$$\delta(i,j) = \begin{cases} 1 \text{ for } i=j \\ 0 \text{ for } i\neq j \end{cases}, \quad (22)$$

q is vector of unknowns

$$\mathbf{q} = \left[\sigma_\mathrm{m}(1),\ldots\,\sigma_\mathrm{m}(N)\right]^T \quad (23)$$

and components of vector **h** are

$$h_i = -\chi_\Delta H_0\,\mathbf{u}_\mathrm{z}\cdot\mathbf{u}_{\mathrm{n}i}. \quad (24)$$

From various possibilities we decided to use method of collocation, showed in figure 7.

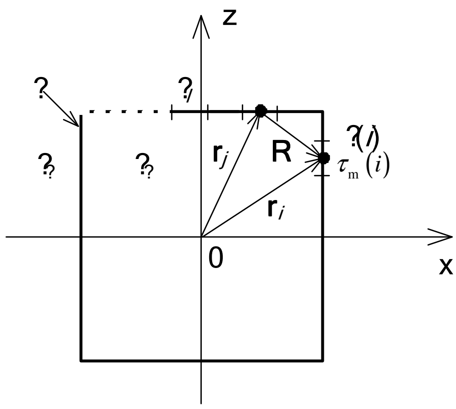

Fig. 7 Collocation method

Effect of element charged surface with surface charge density σ_m is substituted by linear magnetic charge

$$\tau_\mathrm{m} = \sigma_\mathrm{m}\Delta_\mathrm{l} \quad (25)$$

Mentioned method was applied by help of Matlab. Whenever we obtained distribution of magnetic charge surface density **q**, reaction field was solved using

$$\Delta\mathbf{H}(\mathbf{r}_i) = \frac{1}{2\pi}\sum_{j=1}^{N}\sigma_{\mathrm{m}}(j)\frac{\mathbf{u}_{\mathrm{r}}}{R(\mathbf{r}_i,\mathbf{r}_j)} \qquad (26)$$

3. CONCLUSION

The new method was designed for magnetic susceptibility measurement based on MR tomography techniques enables to determine the magnetic susceptibility of such materials, which give no MR signal. Principle of the method was analytically designed and modeled and experimentally verified in laboratory (not presented in this paper).

One of result obtained by numerical modeling using described method is presented below. In this simulation the aluminium specimen (Ω_1) was considered with $\chi_{\mathrm{m1}} = 22.10^{-6}$, length of specimen $z = 20$ mm, thickness $a = (3, 5$ and $7)$ mm. Specimen was immersed into the water with $\chi_{\mathrm{m2}} = -9.10^{-6}$ - (Ω_2). Curve of magnetic intensity is in figure 8, curve of flux density is in figure 9. These curves correspond to theoretical shapes from figure 2.

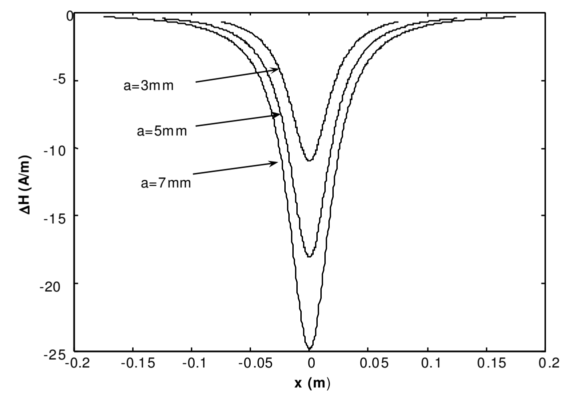

Fig. 8 Intensity of reaction magnetic field simulated by Matlab using collocation method

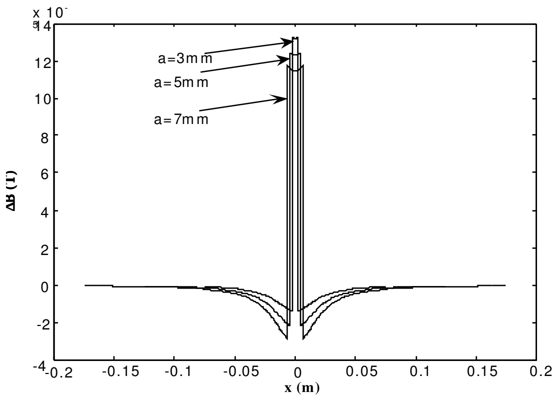

Fig. 9 Flux density of reaction magnetic field simulated by Matlab using collocation method

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