

## POWER ANALYSIS OF TRACTION TRANSFORMER UNDER NON-SINUSOIDAL CONDITIONS

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**Summary** Article deals with power analysis of traction transformer 110/27 kV,  $S_n = 10$  MVA under non-sinusoidal conditions. The power analysis is evaluated by means of IEEE Trial Use Standard Definitions of the Measurement of Electric Power Quantities Under Non-Sinusoidal Conditions, Document Number: IEEE 1459-2000.

**Abstrakt** V článku je prezentována výkonová analýza trakčního transformátoru 110/27 kV,  $S_n = 10$  MVA s neharmonickými průběhy veličin. Výkonová analýza je posuzována v souladu s doporučením standardu IEEE 1459-2000.

### 1. INTRODUCTION

Measured voltage and current waveforms (Fig. 1, Fig. 2) of primary and secondary windings of single-phase transformer are alternating non-harmonic periodic functions, whose can be expressed by means of Fourier series as the sum of harmonic functions

$$v(t) = \sum_{k=1}^{\infty} V_{m(k)} \sin(k\omega t \pm \psi_{(k)}).$$

Waveform distortion  $v_z(t)$  is defined by difference between measured waveform  $v(t)$  and its first harmonic function  $v_1(t)$

$$v_z(t) = v(t) - v_{(1)}(t).$$

The waveform distortion is unambiguously represented by its root mean square value

$$V_z = \sqrt{V^2 - V_{(1)}^2}.$$

Quotient of rms distortion waveform  $V_z$  and rms first harmonic function is called total harmonic distortion (THD)

$$\text{THD V} = 100 \frac{V_z}{V_{(1)}} = 100 \sqrt{\frac{V^2 - V_{(1)}^2}{V_{(1)}^2}} = 100 \sqrt{\frac{V^2}{V_{(1)}^2} - 1}.$$

The total harmonic distortion THD is a suitable tool for monitoring, whereas waveform distortion and harmonic analysis are commonly used at full analysis of the periodic signals.

When we divide rms distortion waveform of the primary winding  $V_{z1}$  by rms distortion waveform of the secondary winding  $V_{zII}$  we obtain

$$\frac{V_{z1}}{V_{zII}} = \frac{V_{(1)I} \cdot \text{THD } V_I}{V_{(1)II} \cdot \text{THD } V_{II}} \Rightarrow \frac{V_{z1}}{V_{zII}} \cdot \frac{\text{THD } V_{II}}{\text{THD } V_I} = \frac{V_{(1)I}}{V_{(1)II}}.$$

### 2. POWER ANALYSIS

The unambiguous information about a change of the state any mass indicates power waveform (Fig. 1c, 2c) –

product of voltage and current waveform

$$p = ui.$$

Power waveform is defined by equation [1,2]

$$\begin{aligned} p &= ui = (u_1 + u_z)(i_1 + i_z) = u_1 i_1 + u_1 i_z + u_z i_1 + u_z i_z = \\ &= u_1 i_1 + p_z = p_1 + p_z, \end{aligned}$$

one is algebraic sum of first harmonic power waveform  $p_1$  and power waveform distortion  $p_z$ . The average value of the power waveform over period  $T$  is called active power

$$\begin{aligned} P &= \frac{1}{T} \int_0^T p \cdot dt = \\ &= \frac{1}{T} \left[ \int_0^T u_1 i_1 \cdot dt + \int_0^T u_1 i_z \cdot dt + \int_0^T u_z i_1 \cdot dt + \int_0^T u_z i_z \cdot dt \right] = \\ &= P_1 + 0 + 0 + P_z, \end{aligned}$$

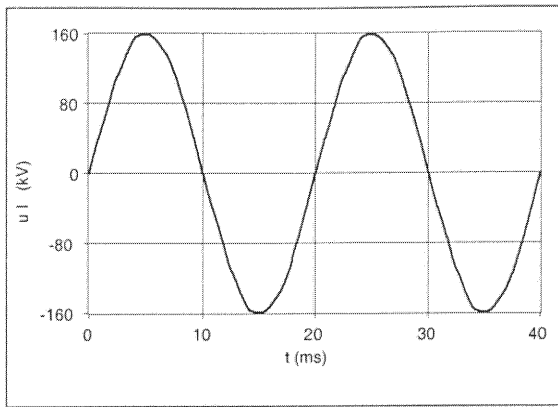
because scalar product of instantaneous values distortion and first harmonic quantity over period is orthogonal [3].

The active power  $P$  is the algebraic sum of the active power of first harmonic voltage and first harmonic current  $P_1$  and active power of distortion voltage and distortion current  $P_z$ , whereas apparent power  $S$  is the geometric sum of the apparent power of first harmonic voltage and first harmonic current  $S_1$  and apparent power distortion  $S_z$

$$\begin{aligned} S &= UI = \sqrt{(U_1^2 + U_z^2)(I_1^2 + I_z^2)} = \sqrt{S_1^2 + S_z^2} = \\ &= \sqrt{U_1^2 I_1^2 + U_1^2 I_z^2 + U_z^2 I_1^2 + U_z^2 I_z^2} = \sqrt{U_1^2 I_1^2 + S_z^2}. \end{aligned}$$

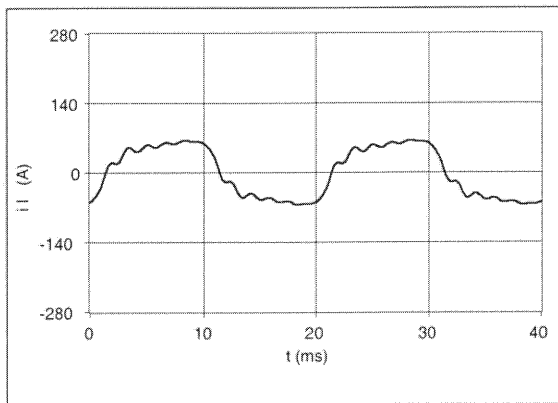
Total harmonic distortion of the power is defined by equation

$$\text{THD S} = 100 \frac{S_z}{S_1} = 100 \sqrt{\frac{S^2 - S_1^2}{S_1^2}} = 100 \sqrt{\frac{S^2}{S_1^2} - 1}.$$



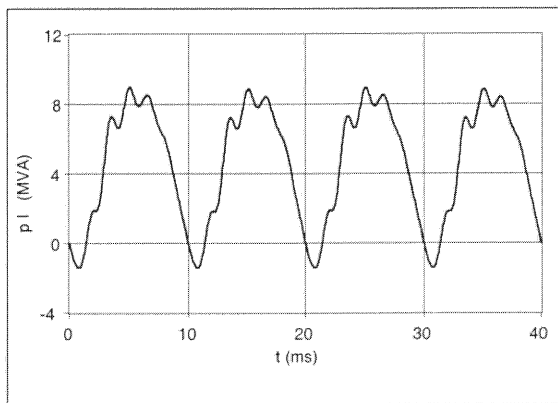
a) voltage waveform

$U_I = 113,7 \text{ kV}$  ; THD  $U_I = 0,5\%$



b) current waveform

$I_I = 50,23 \text{ A}$  ; THD  $I_I = 24,6 \%$

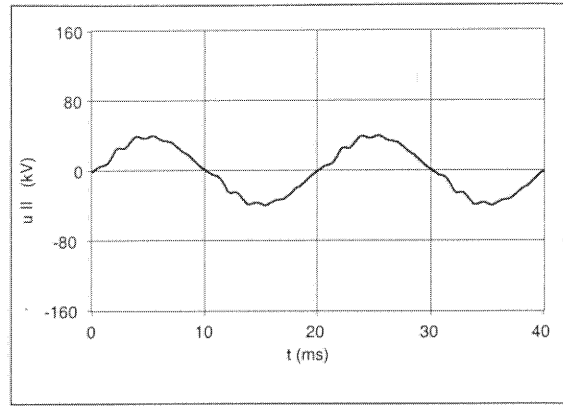


c) power waveform

$S_I = 5,712 \text{ MVA}$  ; THD  $S_I = 24,6 \%$

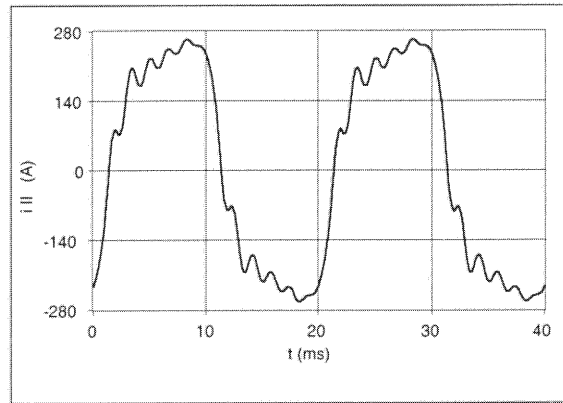
$\lambda_{p I} = 0,807$  .

Fig. 1. The waveforms of the primary winding.



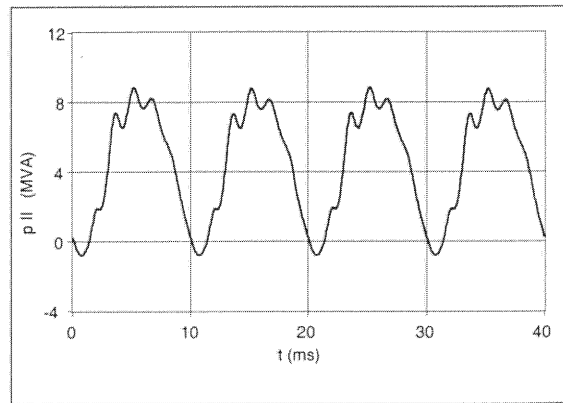
a) voltage waveform

$U_{II} = 26,81 \text{ kV}$  ; THD  $U_{II} = 7,4\%$



b) current waveform

$I_{II} = 203,2 \text{ A}$  ; THD  $I_{II} = 23,6 \%$

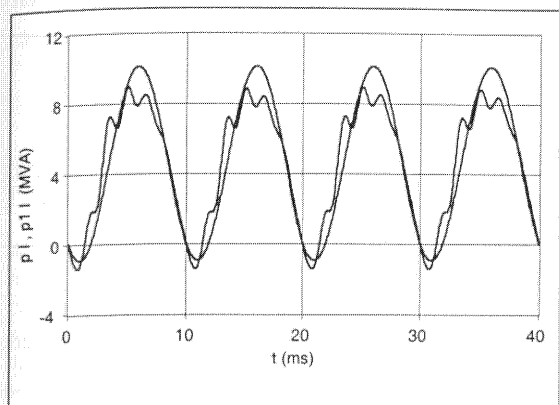


c) power waveform

$S_{II} = 5,448 \text{ MVA}$  ; THD  $S_{II} = 25,4 \%$

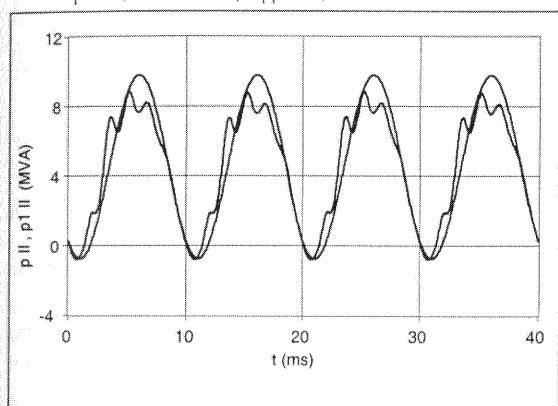
$\lambda_{p II} = 0,828$  .

Fig. 2. The waveforms of the secondary winding.



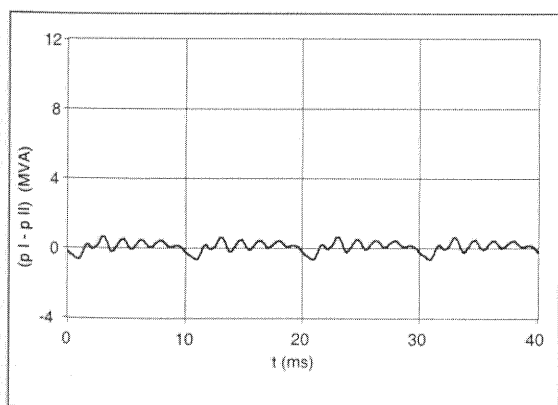
a) power waveform and product of first harmonic voltage and current of primary winding

$$P_I = 4,612 \text{ MW} ; P_{1I} = 4,620 \text{ MW}$$



b) power waveform and product of first harmonic voltage and current of secondary winding

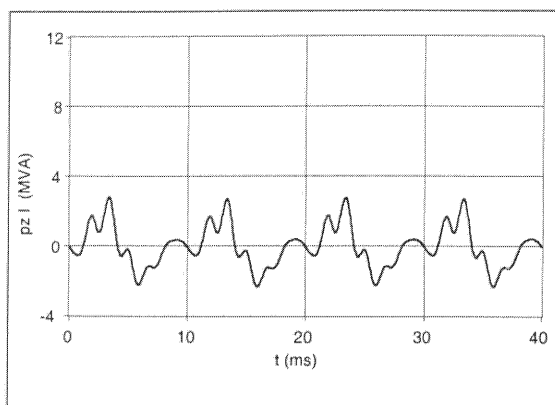
$$P_{II} = 4,511 \text{ MW} ; P_{1II} = 4,525 \text{ MW}$$



c) difference between power waveform of primary and secondary winding

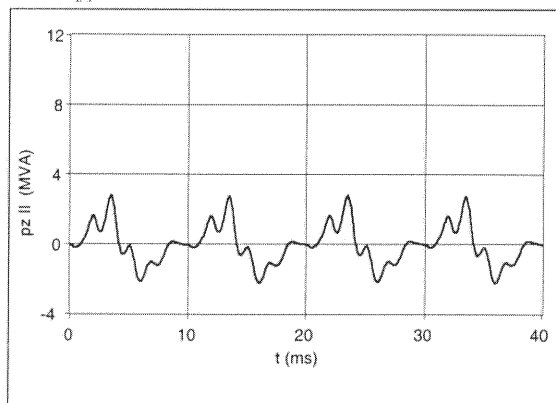
$$P_{\Delta} = 0,101 \text{ MW} ; P_{1\Delta} = 0,095 \text{ MW}$$

Fig. 3. Power waveforms.



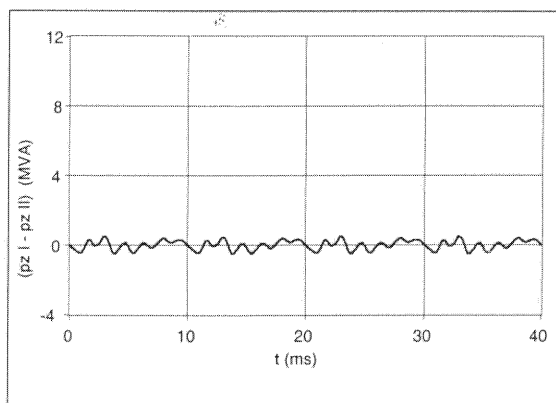
a) power waveform distortion of primary winding

$$P_{zI} = -0,008 \text{ MW}$$



b) power waveform distortion of secondary winding

$$P_{zII} = -0,014 \text{ MW}$$



c) difference between power waveform distortion of primary and secondary winding

$$P_{z\Delta} = 0,006 \text{ MW}$$

Fig. 4. Power waveform distortions.

Instantaneous power loss  $p_{\Delta}$  (Fig. 3c) is defined as algebraic difference between instantaneous power of the primary winding  $p_1$  and instantaneous power of the secondary winding  $p_{II}$

$$p_{\Delta} = p_1 - p_{II},$$

their average values are called active power losses

$$P_{\Delta} = \frac{1}{T} \int_0^T p_{\Delta} \cdot dt = \frac{1}{T} \int_0^T (p_1 - p_{II}) \cdot dt = P_1 - P_{II}.$$

For the full power analysis in accordance with [1,4] we express the instantaneous powers as the algebraic sum of first harmonic power waveform and power waveform distortion by means of equations.

$$p_{\Delta} = p_1 - p_{II} = (p_{11} + p_{z1}) - (p_{1II} + p_{zII}) =$$

$$= p_{11} - p_{1II} + p_{z1} - p_{zII} = p_{1\Delta} + p_{z\Delta},$$

$$p_{1\Delta} = p_{11} - p_{1II},$$

$$p_{z\Delta} = p_{z1} - p_{zII}$$

and active powers analogously

$$P_{\Delta} = P_1 - P_{II} = P_{1\Delta} + P_{z\Delta}$$

$$P_{1\Delta} = P_{11} - P_{1II},$$

$$P_{z\Delta} = P_{z1} - P_{zII}.$$

Figured voltage, current, power time courses (Fig. 1, Fig. 2, Fig. 3, Fig. 4) are characterized by their parameters. Power factor is given by the formula

$$\lambda = \frac{P}{S}.$$

### 3. SIGNIFICANT RESULTS OF POWER ANALYSIS

a) In this case, the active power loss of the traction transformer  $P_{\Delta} = 101 \text{ kW}$  that is 2,2 % from active power  $P_1 = 4,612 \text{ MW}$ .

b) The active power loss of the traction transformer  $P_{\Delta} = 101 \text{ kW}$  is the algebraic sum of the active power loss of the first harmonic voltage and current waveform  $P_{1\Delta} = 95 \text{ kW}$  and active power loss distortion waveform  $P_{z\Delta} = 6 \text{ kW}$ .

c) The values of active powers  $P_1 = 4,612 \text{ MW}$ ,  $P_{II} = 4,511 \text{ MW}$  are less than the values of active powers  $P_{11} = 4,620 \text{ MW}$ ,  $P_{1II} = 4,525 \text{ MW}$  of the first harmonic voltage and current waveform both windings.

d) Active power distortion waveform of the secondary windings  $P_{zII} = -14 \text{ kW}$  and active power distortion waveform of the primary windings  $P_{z1} = -8 \text{ kW}$  reversal operate, because both possess minus sign.

e) Total harmonic distortion of the primary apparent power  $\text{THDS}_I = 24,6\%$  is less than total harmonic distortion of the secondary apparent power  $\text{THDS}_{II} = 25,4\%$ .

### 4. CONCLUSION

In this article is developed the powerful technique that enable us evaluated electromagnetic phenomena under non-sinusoidal conditions.

### Acknowledgement

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### REFERENCES

- [1] IEEE Trial Use Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Non-Sinusoidal, Balanced, or Unbalanced Conditions. Document Number: IEEE 1459-2000, Institute of Electrical and Electronics Engineers, 01-May-2000, 52
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