Design and Implementation of Takagi-Sugeno Fuzzy Tracking Control for a DC-DC Buck Converter

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Abstract. This paper presents the design and implementation of a Takagi-Sugeno (T-S) fuzzy controller for a DC-DC buck converter using Arduino board. The proposed fuzzy controller is able to pilot the states of the buck converter to track a reference model. The T-S fuzzy model is employed, firstly, to represent exactly the dynamics of the nonlinear buck converter system, and then the considered controller is designed on the basis of a concept called Virtual Desired Variables (VDVs). In this case, a two-stage design procedure is developed: i) determine the reference model according to the desired output voltage, ii) determine the fuzzy controller gains by solving a set of Linear Matrix Inequalities (LMIs). A digital implementation of the proposed T-S fuzzy controller is carried out using the ATmega328P-based Microcontroller of the Arduino Uno board. Simulations and experimental results demonstrate the validity and effectiveness of the proposed control scheme.

Keywords
Arduino board, DC-DC buck converter, T-S fuzzy model.

1. Introduction

DC-DC buck converters are widely used in industrial and home environment (mobile phone, computers, and home appliances). Thanks to their increasingly high efficiency, together with a reduced size, weight and cost, they have held an important place in connections to storage batteries, photovoltaic systems, wind turbines, hybrid systems 1, 2, 3 and 4. However, the control of a buck converter is still a challenging task because such a system exhibits a nonlinear behavior with inherent uncertainties and disturbances. Thus, the linear control schemes cannot ensure satisfactory performances over a wide operating range 5. To address this problem, nonlinear and advanced control design methods have been proposed, such as linearization control 6 and 7, sliding mode 8 and 9, adaptive control 10 and 11, backstepping control approach 12 and exact linearization methods 13.

The simplicity of design and the robustness of the sliding mode controller have made it the most used one 14 and 15. But, these advantages are neutralized by the presence of an undesirable phenomenon known as 'chattering' (oscillations having finite frequency and amplitude) which is considered as the main obstacle for implementation 16 and 17. In 18 and 19 an adaptive backstepping controller is proposed for DC-DC buck converters. But it also suffers from difficulties and limitations during the implementation stage. In 20 and 21, a state feedback exact linearization method is applied to DC-DC buck converters. However, in 20, the effects of the different parasitic elements are not taken into account, while in 21 the developed controller cannot be used in a wide range of variation because of the excessive output voltage overshoot.

In the field of fuzzy logic, much research has devoted to the application of fuzzy Mamdani controllers to converters 22 and 23. These works are typically based on a small signal model using the state space averaging method; the model obtained by these methods is only useful for small variations around a specific operating point, whereas the application of Takagi-Sugeno (T-S) fuzzy models on buck converters is not sufficiently investigated. The T-S fuzzy models owe their popularity to their effectiveness in modeling and con-
trolling nonlinear systems [24] and [25]. The T-S fuzzy model represents a nonlinear system by a set of fuzzy IF-THEN rules, which locally represents the input-output relationships of a system expressing each conclusion with a linear subsystem. The advantage of this type of fuzzy model lies in the stability of the fuzzy system which can be analyzed using the Lyapunov method, and then treated in terms of the feasibility of a set of Linear Matrix Inequalities (LMIs). In this case, the problem can be solved easily by numerical convex optimization techniques [26].

Recently, the T-S fuzzy tracking control problem has been addressed in many works based on a concept known as Virtual Desired Variables (VDVs) to simplify the development of the control law and reference model [27], [28], [29], [30], [31] and [32]. Furthermore, the VDVs concept allows converting easily the tracking problem into a stabilization one. The concept has been analyzed in a number of studies using the Lyapunov approach and successfully investigated in many control applications. For example, in [28], it was employed to control a wind energy conversion system. In [29], it was employed to control a Permanent Magnet Synchronous Machine (PMSM) while in [30], the same concept was used to control a photovoltaic system. In [31], it was used to develop a fuzzy torque observer for PMSM and in [32], it was combined with H infinity performance to design a T-S fuzzy controller for a PMSM.

In this paper, the purpose is to develop a T-S fuzzy tracking controller for a buck converter based on the VDVs concept. In this case, the proposed controller can be used to drive the system to follow the desired reference. First, the nonlinear buck converter system is represented by a T-S fuzzy model. Then, a fuzzy controller is developed based on a set of virtual desired variables to simplify the design of the reference model and control law. Next, the tracking performance of the enhanced fuzzy system is analyzed by the Lyapunov method which can be formulated into LMIs problems. Simulation tests are performed on a buck converter to verify the controller’s efficiency. Finally, the proposed controller is implemented on an ATmega328P-based microcontroller of Arduino Uno.

The remainder of this paper is composed as follows: the Sec. 2 is devoted to details on the mathematical buck converter model. In the Sec. 3, the proposed fuzzy control method is introduced. It consists of three main blocks: the first part deals with the T-S fuzzy controller, the second part is dedicated to stability analysis conditions and the third one deals with the determination of the desired reference model and the nonlinear tracking controller. The results of the simulation and practical implementation of the proposed controller are given in Sec. 4 and Sec. 5 followed by a conclusion at the end of this work.

2. Mathematical Buck Converter Model

The buck converter can be represented by the following nonlinear state space system form:

\[
\begin{align*}
\dot{x}(t) &= f(x(t)) + g(x(t))u(t) + \eta, \\
g(t) &= \varphi(x(t)) \\
x(t) &= \begin{bmatrix} i_L(t) \\
v_o(t) \end{bmatrix}, \\
\eta &= \begin{bmatrix} \frac{V_0}{2} \\
0 \end{bmatrix}, \\
f(x(t)) &= \begin{bmatrix} R\frac{L}{(R+R_C)}i_L(t) - \frac{R}{L}\frac{i_L}{(R+R_C)}v_o(t) \\
\frac{1}{C(R+R_C)}i_L(t) - \frac{1}{C(R+R_C)}v_o(t) \end{bmatrix}, \\
g(x(t)) &= \begin{bmatrix} \frac{R_i}{(R+R_C)}i_L(t) + \frac{R}{R+R_C}v_o(t) \end{bmatrix}, \\
\varphi(x(t)) &= \begin{bmatrix} \frac{R_i}{(R+R_C)}i_L(t) + \frac{R}{R+R_C}v_o(t) \end{bmatrix},
\end{align*}
\]

where \(R_i\) is the resistance of the transistor (MOSFET), \(R_L\) is the winding resistance of the inductor, \(V_D\) is the threshold voltage of the diode, \(R_C\) is the equivalent series resistance of the filter capacitor and \(V_{in}\) is the input voltage. \(i_L\), \(v_o\) and \(u\) represent, respectively, the inductance current, the output voltage and the duty cycle of the buck converter, as shown in Fig. 1. It should be mentioned that the internal resistance \(R_m\), the diode’s forward voltage \(V_D\), the equivalent series resistance of the filter capacitor \(R_C\) and the winding resistance of inductor \(R_L\) have not been taken into account in many previous researches, which can perturb the control of the buck converter system. Consequently, this paper considers a more general case.

3. Fuzzy Control Design

The goal of this paper is to develop a fuzzy controller that permits to pilot the states of a buck converter \(x = \begin{bmatrix} i_L \\
v_o \end{bmatrix}^T\) to track a desired trajectory \(x_d = \begin{bmatrix} i_{Ld} \\
v_{od} \end{bmatrix}^T\). Firstly, we design a fuzzy controller based on the T-S fuzzy model of a buck converter system and then develop a virtual reference...
model and nonlinear controller according to the desired output voltage. Thus, the fuzzy tracking control scheme shown in Fig. 2 is proposed.

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + B(i_L)u(t) + E, \\
y(t) &= v_o = Cx(t) 
\end{align*}
\]

where \( F_{11} \) and \( F_{12} \) are the membership functions given by:

\[
F_{11}(i_L) = \frac{i_L(t) - \bar{i}_L}{\bar{i}_L - \underline{i}_L}, \quad F_{12}(i_L) = 1 - F_{11}(i_L). \quad (10)
\]

The sub-matrices are defined as:

\[
A_1 = A_2 = \begin{bmatrix} -\frac{1}{T} \left( R_L + \frac{R R_C}{R + R_C} \right) - \frac{R}{C(R + R_C)} & -\frac{R}{C(R + R_C)} \\ \frac{1}{T} \left( V_{in} + V_D - R_M i_L \right) & 0 \end{bmatrix}, \quad (11)
\]

\[
B_1 = \begin{bmatrix} \frac{1}{T} \left( V_{in} + V_D - R_M i_L \right) & 0 \\ 0 & 0 \end{bmatrix}, \quad (12)
\]

\[
B_2 = \begin{bmatrix} 0 & 0 \end{bmatrix}. \quad (13)
\]

The final output of fuzzy model is inferred as follows:

\[
\dot{x}(t) = \sum_{i=1}^{n} h_i(z(t)) (A_i x(t) + B_i u(t) + E), \quad (14)
\]

where \( h_i(z) = \omega_i(z)/\sum_{i=1}^{n} \omega_i(z), \omega_i(z) = \prod_{j=1}^{m} F_{ij}(z_j) \) for all \( t > 0, h_i(z) > 0 \) and \( \sum_{i=1}^{n} h_i(z) = 1 \).

3.2. Control Design and Stability Analysis

The T-S fuzzy control is necessary to satisfy the following condition:

\[
x(t) - x_d(t) \rightarrow 0 \quad \text{as} \quad t \rightarrow \infty, \quad (15)
\]

where \( x_d(t) \) represents the desired trajectory variable. Let us define the tracking error as \( \tilde{x}(t) = x(t) - x_d(t) \). Then, its time derivative can be given by:

\[
\dot{\tilde{x}}(t) = \dot{x}(t) - \dot{x_d}(t). \quad (16)
\]

By substituting Eq. (14) into Eq. (16) and adding the term \( \sum_{i=1}^{n} h_i(z(t)) (A_i \tilde{x}(t) - \dot{x_d}(t)) \), Eq. (16) becomes:

\[
\dot{\tilde{x}}(t) = \sum_{i=1}^{r} h_i(z) (A_i \tilde{x}(t) + B_i u(t) + E_i) - \dot{x}_d. \quad (17)
\]

In Eq. (17), let us choose a new control variable \( \tau(t) \) that satisfies the following condition:

\[
\begin{align*}
\sum_{i=1}^{r} h_i B_i \tau &= \sum_{i=1}^{r} h_i (z(t)) (A_i x_d + B_i u + E_i) - \dot{x}_d \quad (18)
\end{align*}
\]

By using Eq. (18), the tracking error derivative Eq. (17) can be rewritten as follows:

\[
\dot{\tilde{x}}(t) = \sum_{i=1}^{r} h_i(i_L(t))(A_i \tilde{x}(t) + B_i \tau(t)). \quad (19)
\]
The new controllers are developed to deal with the fuzzy tracking control problem as follows:

- **Controller rule 1**: If $i_L(t)$ is $F_{11}$ Then $\tau(t) = -K_1 \dot{x}(t)$,
- **Controller rule 2**: If $i_L(t)$ is $F_{12}$ Then $\tau(t) = -K_2 \dot{x}(t)$.

The final fuzzy controller output is given by:

$$\tau(t) = -\sum_{i=1}^{r} h_i(z(t))K_i \dot{x}(t). \quad (20)$$

By substituting Eq. (20) into Eq. (19), the final closed-loop system takes the following form:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t))h_j(z(t))(A_i - B_jK_j) \dot{x}(t). \quad (21)$$

By letting $G_{ij} = (A_i - B_jK_j)$, Eq. (21) can be rewritten as follows:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t))h_j(z(t))G_{ij} \dot{x}(t). \quad (22)$$

**Stability Analysis.** Obtaining the fuzzy controller consists in determining the gains $K_i$ satisfying the conditions of the following theorem [28] and [29]:

**Theorem 1** The continuous model Eq. (22) is asymptotically stable via the fuzzy controller Eq. (20), if there exists a diagonal matrix $D$, matrices $Q_{ij}$ with: $Q_{ii} = Q_{ii}^T$ and $Q_{ij} = Q_{ji}^T$ for $i \neq j$, and a common positive definite matrix $P > 0$ such that:

$$G_{ii}^T P + PG_{ii} + Q_{ii} + DPD < 0, \quad i = 1, \ldots, r, \quad (23)$$

$$\left(\frac{G_{ij} + G_{ji}}{2}\right)^T P + P \left(\frac{G_{ij} + G_{ji}}{2}\right) + Q_{ij} < 0, \quad i < j \leq r, \quad (24)$$

$$\begin{bmatrix}
Q_{11} & Q_{12} & \cdots & Q_{1r} \\
Q_{12} & Q_{22} & \cdots & Q_{2r} \\
\vdots & \vdots & \ddots & \vdots \\
Q_{1r} & Q_{2r} & \cdots & Q_{rr}
\end{bmatrix} \equiv \bar{Q} > 0, \quad (25)$$

for $i, j = 1, \ldots, r$, s.t. the pairs $(i, j)$ such that: $h_i(z)h_j(z) = 0, \forall t$.

The determination of the fuzzy control gains requires changing the conditions of the previous theorem into an equivalent problem of linear matrix inequalities. This transformation corresponds to simple objective changes of variables $X = P^{-1}K_i = M_i X^{-1}$ and the use of a congruence in inequalities Eq. (23), Eq. (24) and Eq. (25). Then, the following LMIs can be obtained:

$$\exists X \in X^T > 0, \exists Y_{ii} = Y_{ii}^T, \exists Y_{ij} = Y_{ji}^T, \exists M_i:$$

$$\begin{bmatrix}
X A_i^T + A_i X + X A_j^T + A_j X - B_i M_i - M_i^T B_i^T - B_j M_i - M_j^T B_j^T + 2Y_{ij} X D^T \\
- X
\end{bmatrix} < 0, \quad (26)$$

$$\begin{bmatrix}
Y_{11} & Y_{12} & \cdots & Y_{1r} \\
Y_{12} & Y_{22} & \cdots & Y_{2r} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{1r} & Y_{2r} & \cdots & Y_{rr}
\end{bmatrix} \equiv \bar{Y} > 0. \quad (27)$$

3.3. Desired Reference Model and Nonlinear Controller

In order to determine the desired reference model $x_d$ and nonlinear controller $u(t)$, we use Eq. (18) which can be rewritten as follows:

$$\sum_{i=1}^{r} h_i B_i (u - \tau) = -\sum_{i=1}^{r} h_i A_i x_d - \sum_{i=1}^{r} h_i E + \dot{x}_d. \quad (29)$$

Noting that:

$$A = \sum_{i=1}^{r} h_i A_i, \quad B = \sum_{i=1}^{r} h_i B_i, \quad E = \sum_{i=1}^{r} h_i E_i. \quad (30)$$

Then, Eq. (29) can be rewritten in the following form:

$$B(u(t) - \tau(t)) = -Ax_d(t) - E + \dot{x}_d(t). \quad (31)$$

Eq. (31) can be rewritten as follows:

$$\begin{bmatrix}
\frac{1}{T}[V_{in} + V_D + R M i(t)] \\
0
\end{bmatrix} (u(t) - \tau(t)) =$$

$$= -\begin{bmatrix}
\frac{1}{T} [RL + \frac{RR_E}{R + RC}] - \frac{R}{C(R + RC)} \\
\frac{R}{C(R + RC)}
\end{bmatrix} \begin{bmatrix}
i_{ld} \\
v_{cd}
\end{bmatrix} +$$

$$\begin{bmatrix}
- \frac{V_{in}}{T} \\
0
\end{bmatrix} + \begin{bmatrix}
i_{ld} \\
v_{cd}
\end{bmatrix}. \quad (32)$$

It should be mentioned that the nonlinear control law and the desired reference model will be calculated according to the desired output voltage.

From the second equation of Eq. (32) we obtain

$$-\frac{R}{C(R + RC)} x_{ld} + \frac{1}{C(R + RC)} x_{2d} = 0$$

$$\Rightarrow x_{ld} = \frac{x_{2d}}{R}. \quad (33)$$

From the first equation of Eq. (32) we can derive the nonlinear control law $u(t)$, as follows:

$$u(t) = \frac{(B_{ld} + \frac{RC}{R + RC} + \frac{R}{RC}) x_{2d} + V_D}{V_{in} + V_D + R M i(t)}. \quad (34)$$
4. Simulation Results

In order to verify the performance of the proposed fuzzy tracking control, simulation test was carried out on a DC-DC buck converter. The controller gains are obtained by solving the LMIs Eq. (26), Eq. (27) and Eq. (28), as follows:

\[
K_1 = \begin{bmatrix}
0.4829 & 0.1582 \\
0.4537 & 0.1345
\end{bmatrix}, \quad (35)
\]

\[
K_2 = \begin{bmatrix}
0.4537 & 0.1345
\end{bmatrix}. \quad (36)
\]

The first simulation is carried out in output voltage reference \( v_{od} = 8 \) V. The responses of the inductance current, output voltage, PWM signal and duty cycle are depicted in Fig. 3(a), Fig. 3(b), Fig. 3(c) and Fig. 3(d), respectively. The results indicate that the steady states track the desired trajectories perfectly. It is also shown that the time response required to follow the reference model is very short (0.0012 s). It can be concluded that the proposed T-S fuzzy control has a good tracking performance. In the second simulation test, the T-S fuzzy controller is compared with the base-line PI (Proportional Integrator) controller due to its popularity. The PI parameters \((K_p \text{ and } K_i)\) are calculated based on the following buck transfer function

\[
c(t) = \frac{1}{LC} \int i(t) dt \quad \text{and} \quad v(t) = \frac{1}{C} \int i(t) dt
\]
\[ G(s) = \frac{V_o}{\mu} = V_{in} \left( \frac{R}{R + R_L} \right) \left( \frac{s}{\Omega(s)} + 1 \right), \quad (37) \]

where
\[ \Omega(s) = \frac{s^2}{\omega_0^2} + \frac{s}{Q\omega_0} + 1, \quad (38) \]
\[ \omega_0 = \frac{1}{\sqrt{LCR + Rc}} \quad \omega_{ZERO} = \frac{1}{CR_c}, \quad (39) \]
\[ Q = \frac{1}{\omega_0 \left( \frac{L}{R + R_L} + \frac{RR_LC}{R + R_L} + R_cC \right)}. \quad (40) \]

The PI parameters \((K_p, K_i)\) are obtained by using the known compensation method, as follows:
\[ K_p = 0.195, \quad K_i = 9.88. \quad (41) \]

The response of the output voltage for \(v_{od} = 5\) V rise time, the settling time and the overshoot for the two methods.

From Fig. 4 and Tab. 1, it can be confirmed that the T-S controller offers superior performance and fast dynamic response in terms of rapidity and limitation of overshoot.

5. Experimental Verification

To verify the simulation results, a special package known as input/output (I/O) support package is used with Matlab environment, which has been designed by MathWorks for microcontroller-based Arduino board to interface the Matlab/Simulink with the hardware setup without any programming language [34]. This package is primarily used for real-time communication between Arduino board and Simulink, as shown in Fig. 5. The hardware consists of a buck converter, an Arduino Uno, a voltage sensor and a current sensor. The hardware and Simulink model used in this implementation stage are shown in Fig. 6(a) and Fig. 6(b), respectively. Note that the parameters given in Appendix A are considered for the development of a buck converter prototype.

The experiment is conducted with a multi-step voltage reference. The experimental waveform and simulation response of output voltage are illustrated in Fig. 8(a) and Fig. 8(b). It is obvious that the obtained results in both the simulation and the implementation are in good agreement.

After the previous verification and validation of the simulation results, the proposed system should be created as a standalone project that does not need to be
connected to the Simulink (host computer) [35]. In this respect, it is necessary to deploy the proposed tracking control algorithm to the Arduino board. To achieve this, an Arduino support package is used, as shown in Fig. 7. The experimental waveforms of the output voltage and the duty cycle for a reference voltage $v_{od} = 5$ V are shown in Fig. 9.

These practical results demonstrate that buck converter can be controlled by the proposed method to flow exactly the desired output voltage.

![Simulation results of output voltage.](image-a)

(a) Simulation results of output voltage.

![Experimental waveform of output voltage.](image-b)

(b) Experimental waveform of output voltage.

Fig. 8: Simulation and experimental results for multi-step voltage reference.

![Experimental results for desired voltage $v_{od} = 5$ V.](image-c)

Fig. 9: Experimental results for desired voltage $v_{od} = 5$ V.

6. Conclusion

A T-S fuzzy tracking controller is proposed for a DC-DC buck converter which is able to pilot the system to follow a desired reference model. The controller gains are obtained based on sufficient conditions formulated into LMIs form and solved using optimization tools. Experimental and simulation results show that the buck converter can be controlled effectively at different operating regions by the proposed method and can overcome the limitations of classical controllers.

References


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Abderrezak METATLA received the Engineer, Magister and Ph.D. degrees in electromechanical engineering from Badji Mokhtar University in Annaba, Algeria in 2004, 2009. Since 2004, he has been in the Electromechanical Department of Skikda University, Algeria where he is currently a Professor of electrical engineering. His main research fields include electrical machines, control systems and renewable energy.

Appendix A

Buck Converter Parameters

<table>
<thead>
<tr>
<th>Circuit parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOSFET resistance $R_{m}$</td>
<td>0.1 Ω</td>
</tr>
<tr>
<td>Threshold voltage of the diode $V_D$</td>
<td>0.8 V</td>
</tr>
<tr>
<td>Output capacitor $C$</td>
<td>270 μF</td>
</tr>
<tr>
<td>Output capacitor resistance $R_c$</td>
<td>0.18 Ω</td>
</tr>
<tr>
<td>Inductor $L$</td>
<td>600 μH</td>
</tr>
<tr>
<td>Winding resistance of inductor $R_L$</td>
<td>0.1 Ω</td>
</tr>
<tr>
<td>Resistance load $R$</td>
<td>30 Ω</td>
</tr>
<tr>
<td>Input voltage $V_{in}$</td>
<td>10 V</td>
</tr>
<tr>
<td>Switching frequency $f_s$</td>
<td>31.38 kHz</td>
</tr>
</tbody>
</table>