Optimization of Nonuniform Linear Antenna Array Topology

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Abstract. *This paper deals with the use of a Nonuniform Linear Antenna array (NLA) for determining the Directions of Arrival (DOA) of a signal in 2450 MHz frequency band. First, the principle of the DOA estimation method is described for the case of the MUSIC method. This paper also discusses the possibilities of optimizing the position of antenna elements in the NLA configuration, which are performed in analytical solutions and simulations. The simulation results are compared to the analytical results to obtain optimal NLA configurations for determining the signal DOA. Simulation results show that the probability of resolution and accuracy in determining the signal DOA are dependent on the antenna array aperture essentially. Furthermore, the realized NLA configuration was verified by an experimental measurement. The obtained experiment results demonstrate that the applied MUSIC method for NLA configuration is suitable and also highly accurate in determining the signal DOA, which was verified.*

Keywords

Determining the direction of arrival, nonuniform linear antenna array, optimization.

1. Introduction

Direction of Arrival (DOA) estimation is an important part of the signal processing field. The processing of signals from antenna arrays possesses a lot of benefits compared to signal processing from only one antenna element. Antenna arrays are used mainly for addressing the following problems: noise reduction, locating multiple signal sources, estimating the number of signal sources etc. Antenna array signal processing is widely used in many areas of interest, such as radar, sonar, communication, seismology, medical, etc. [1], [2] and [3].

In military applications, the direction of arrival of radar signals is one of the most interesting parameters in Electronic Intelligence (ELINT). These DOA estimations are used in many classical methods such as amplitude methods and phase interferometers. Using these methods, one can measure the DOA with appropriate accuracy only for one signal source at a time. However, in real life, it is often needed to determine the DOA of more than one signal or to address signal reflection problems. In these cases, the DOAs determined by the classical methods are inaccurate or highly distorted. One of the main requirements in either civilian or military applications is the DOA estimation of the Wi-Fi 802.11 b/g signal sources. As the frequency band is being shared by many transmitters, the classical methods such as amplitude or interferometric failed. One way of solving this problem is to use the high resolution sub-space methods of DOA estimation, mentioned in this paper.

DOA estimation methods which use antenna arrays can be divided into three groups: conventional methods, sub-space methods and maximum likelihood methods. The subspace methods are quite reliable with very good properties and there are other subspace-based methods with various efficiencies (for example MUSIC, Root MUSIC, ESPRIT, Minimum Norm method etc. [3] and [4]). This paper is focused on the MUSIC method (Multiple Signal Classification), which is a typical subspace method of DOA estimation. The MUSIC method falls under the High Angle Resolution (HAR) class.

When using the MUSIC method, the DOA estimations are affected not only by the antenna array configuration, but also by other parameters: Signal Noise Ratio (SNR), the number of antenna elements, the number of signal samples, antenna array length, the cross-correlation of arriving signals, etc. These methods can be used with 1-D, 2-D or 3-D antenna array configurations, however, they are used mainly for uniform arrays, for example the Uniform Linear Array (ULA) and Uniform Circular Array (UCA). There are some ways of increasing the accuracy and DOA resolution - such as increasing the number of antenna elements and signal samples, although in many cases this is not applicable or practical.

The Nonuniform Linear Array (NLA) was introduced in the 1960s and has been used mainly for antenna beamforming in the active sensing. NLA is used to build antenna arrays that have a larger aperture, while the number of elements is comparable to a ULA. Increasing the spacing antenna elements to more than half of its wavelength leads to a DOA estimation ambiguity due to the similarities in covariation matrix [5]. The simplest way to suppress this ambiguity effect is to use at least one pair of antenna elements with their distance shorter than half of the wavelength. In [6], the criterion for the ambiguity suppression is presented, where the common divisor of each antenna distance is equal to 1. The achieved results of this antenna array configuration were verified by the simulation in [7]. The main advantage of using the NLA antenna array configuration is the option of dealing with special cases where the number of signal DOA sources is higher than the number of antenna array elements [8], [9], [10] and [11].

This paper focuses on the signal source DOA estimation, especially on the NLA configuration optimization, with the aim of increasing its angle accuracy and its resolution. In the second part of this article, a model of antenna array output signal simulation is presented (ULA and NLA modelling), and the MUSIC algorithm is analyzed for NLA application. The third chapter is dedicated to the optimization of the antenna array configuration and the fourth chapter compares cases with three-element and four-element antenna configurations. Practical tests of one signal source with four antenna elements NLA are presented in the fifth chapter. The final chapter summarizes the results and discusses the possible future developments in DOA estimation using NLA.

2. The Antenna Array Signal Output Modelling and MUSIC Method

2.1. The Antenna Array Signal Output Modelling

It is assumed that the antenna linear array is composed of M identical antenna elements with a constant distance d (antenna base).

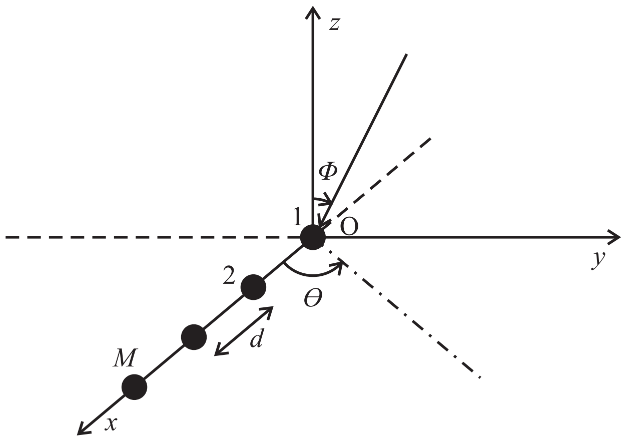

Fig. 1: Linear antenna array and antenna elements in space.

The antenna elements are distributed on the x-axis with a constant distance d as shown in Fig. 1. Corresponding to ULA configurations, the arriving signals in the xy plane are processed and the angles of arrival θ are estimated. For each signal, the elevation angle Φ is equal to $\pi/2$. Assuming that the air has no effect on signal spreading, the only difference between the signals at first antenna element and the second antenna element is the time delay. Phase delay depends only on the distance of antenna elements and the angle of arrival. The first antenna element position is chosen as a reference point of the antenna array, i.e. $(x_1, y_1, z_1) = (0, 0, 0)$. The approaching plane wave arrives to the second antenna element via a path $d \cdot \sin\theta$ that is longer than the distance to the first antenna element. The phase delay of the incoming wave between the first and the second antenna element is $\varsigma_1 = \beta \cdot d \cdot \sin\theta$, where β is wavenumber equal to $2\pi/\lambda$. Similarly, for other antenna elements, the phase delays are $\varsigma_2 = \beta \cdot 2 \cdot d \cdot \sin\theta$, $\varsigma_3 = \beta \cdot 3 \cdot d \cdot \sin\theta$, etc. Finally, the direction vector corresponding to θ is as follows:

$$a(\theta) = \left[1, e^{j\beta d \sin\theta}, \ldots, e^{j\beta(M-1)d\sin\theta}\right]^T. \qquad (1)$$

Assuming that the numbers D are coming from narrowband signal sources ($D < M$), they are described by steering vectors $\vec{S} = (s_1, s_2, \ldots, s_D)$, and correspond to the DOA signals $\theta_1, \theta_2, \ldots, \theta_D$. All the elements of the antenna array are connected to a receiver with the ability to record K signal samples. The incoming signals can be described by the formula:

$$\begin{bmatrix} x_1(k) \\ x_2(k) \\ \cdots \\ x_M(K) \end{bmatrix} = [a(\theta_1) \cdots a(\theta_D)] \begin{bmatrix} s_1(k) \\ s_2(k) \\ \cdots \\ s_D(k) \end{bmatrix} + \mathbf{n}(k). \qquad (2)$$

It is possible to rewrite the Eq. (2) in a vector form:

$$\mathbf{x}(k) = \mathbf{A} \cdot \mathbf{s}(k) + \mathbf{n}(k), \qquad (3)$$

where $\mathbf{x}(k)$ is the received signal matrix, $\mathbf{A}$ is the steering matrix, $\mathbf{s}(k)$ is the incoming signal matrix and $\mathbf{n}(k)$ is the matrix containing the Additive White Gaussian Noise (AWGN) with zero mean level and standard deviation δ_n.

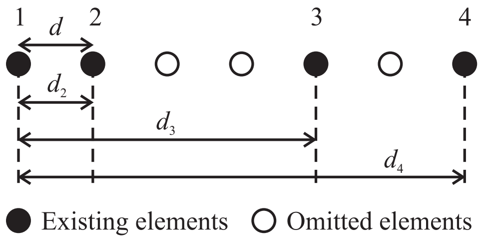

Fig. 2: Nonuniform linear antenna array with 4 antenna elements.

If the model of the output signals is considered a Nonuniform Linear Array (NLA), NLAs can then be principally divided into two groups: noninteger NLA and integer (whole number) NLA. In noninteger NLAs, antenna elements are distributed randomly. These NLAs are described in [8]. In the case of integer NLAs, the antenna elements are distributed on integer multiples with the unit distance $d \leq \lambda/2$ (where λ is a signal wavelength). This paper is focused on the integer NLAs. These antenna arrays are identical to standard ULAs, but some antenna elements are removed (Fig. 2).

Assuming that the first antenna element is a reference or in a reference position ($d_1 = 0$), the other antenna elements are distributed as follows: $d_{NLA} = [0\ d_2\ \ldots\ d_M]$. The output signal formula will be similar to the ULA, and the steering vector can be formed according to the arriving signals as:

$$a(\theta) = \left[1, e^{j\beta d_2 \sin\theta}, \ldots, e^{j\beta d_M \sin\theta}\right]^T. \quad (4)$$

The antenna array output signal matrix can then be described similarly to the NLA in Eq. (2).

2.2. MUSIC Method

The MUSIC method is a sub-space method based on the characteristic structure of the output covariation matrix. The acronym MUSIC stands for *Multiple Signal Classification*. The MUSIC method is an algorithm for parameter estimation providing information about the number of signal sources, their DOAs, their mutual correlations and their noise power levels. It is assumed that the signals are uncorrelated. Then, the covariance matrix of the received signals $\mathbf{R}_{xx}$ can be expressed as:

$$\mathbf{R}_{xx} = E\left[\mathbf{x}\mathbf{x}^H\right] = \mathbf{A}E\left[\mathbf{s}\mathbf{s}^H\right]\mathbf{A}^H + E\left[\mathbf{n}\mathbf{n}^H\right], \quad (5)$$

or

$$\mathbf{R}_{xx} = \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \delta_n^2\mathbf{I},\ \mathbf{R}_{ss} = E\left[\mathbf{s}\mathbf{s}^H\right], \quad (6)$$

where $\mathbf{R}_{xx}$ is the covariance matrix of the transmitted signals, and $[]^H$ is a symbol for the Hermitian conjugated matrix.

For evaluating the correlation matrix, the SVD (Singular Value Decomposition) algorithm can be used to obtain the eigenvalues and eigenvectors.

$$\mathbf{R}_{xx} = \sum_{i=1}^{M} \mu_i \nu_i \nu_i^H. \quad (7)$$

Eigenvalues μ_i of this matrix are real numbers and for real eigenvalues, the following formula applies:

$$\mu_i = \begin{cases} \mu_i + \delta_n^2 & \text{for} \quad i = 1, \ldots, D, \\ \delta_n^2 & \text{for} \quad i = D+1, \ldots, M, \end{cases} \quad (8)$$

where D is the number of eigenvalues that are greater than the noise power level and $M - D$ are approximately equal to the noise power level.

Eigenvectors ν_i form two sub-spaces: the signal subspace and the noise sub-space. The MUSIC method uses the orthogonality between the signal and noise subspaces

$$a^H(\theta_d)\nu_i = 0, \quad i = D+1, \ldots, M; \quad d = 1, \ldots, D, \quad (9)$$

where $a(\theta_d)$ is the signal vector orthogonal to the noise subspace.

Then, the spatial MUSIC spectra can be computed as:

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta)\nu_i\nu_i^H a(\theta)}. \quad (10)$$

While changing the steering angle θ in the determined interval - i.e. in the interval of the possible arrival directions ($-90°$ to $90°$), the DOAs can be extracted as the local maxima of the function $P_{MUSIC}(\theta)$.

3. Optimization of Antenna Element Positions

Many optimization criteria are used for the optimization of antenna element positions. These criteria are affected by the shape of the antenna beam pattern, i.e. by the antenna main beam width and Side Lobe Levels (SLL). In this paper, these criteria will be optimized with CRB (Cramer-Rao Bound) parameter, describing limiting or accessible accuracy of the direction finder, depending on the antenna element positions in the 1-D space or the linear antenna array, respectively. It is well-known that the main problem in the NLA design is the DOA estimation ambiguity.

In the beginning of the optimization process, the ambiguity suppression process is going to be applied in the same way that it is used in a phase interferometer. This criterion is Greatest Common Divisor (GCD) of the antenna base $d_2, \dots, d_M$, which could be equal to 1.

$$\mathrm{GCD}(d_2, \dots, d_M) = 1. \quad (11)$$

The CRB parameter is a useful tool for NLA DOA accuracy estimation evaluation because it characterizes attainable accuracy of the direction finder [3].

After a few mathematical transformations, the following formula can be formed:

$$\mathrm{CRB} = \frac{\delta_n^2}{2K}\left\{\mathrm{Re}\left\{\left[\mathbf{S}_f\left[\left(\mathbf{I}+\mathbf{A}^H\mathbf{A}\frac{\mathbf{S}_f}{\delta_n^2}\right)^{-1}\cdot \left(\mathbf{A}^H\mathbf{A}\frac{\mathbf{S}_f}{\delta_n^2}\right)\right]\right]\circ \mathbf{H}^T\right\}\right\}^{-1}. \quad (12)$$

where $\circ$ is the Hadamard product, $\mathbf{H} = \dot{\mathbf{A}}^H\mathbf{P}_A^{\perp}\dot{\mathbf{A}}$, $\dot{\mathbf{A}} = \frac{\partial \mathbf{A}}{\partial \theta}$ (is a partial derivation of the steering matrix by θ), $\mathbf{P}_A^{\perp}$ is projection onto the noise subspace matrix which is computed by the formula: $\mathbf{P}_A^{\perp} = \left[\mathbf{I} - \mathbf{A}\left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H\right]$; θ are arrival angles of the incoming signals $D(\theta_1), \dots, \theta_D$; $\mathbf{S}_f$ is the covariation matrix of the transmitting signals.

In case of fully uncorrelated signals $\mathbf{S}_f$, the diagonal matrix will be $diag\left(\delta_{s1}^2, \dots, \delta_{sD}^2\right)$, K is the number of signal samples.

The CRB_1 parameter for only one incoming signal $(D = 1)$ can be derived from Eq. (11) as:

$$\mathrm{CRB}_1 = \frac{(1 + M \cdot \mathrm{SNR})}{2K \cdot \mathrm{SNR}^2 \cdot \left(\frac{2\pi}{\lambda}\right)^2 \cos^2(\theta)} \cdot \frac{1}{M^2 \cdot \left[\frac{d_1^2 + \dots d_M^2}{M} - \frac{(d_1 + \dots + d_M)^2}{M^2}\right]}. \quad (13)$$

From Eq. (13), it is apparent that the CRB1 fully depends on: the number of the measured signal samples K, SNR, λ, the signal's DOA and the NLA topology. Assuming that M, K, SNR, λ, and the signal's DOA are fixed, the CRB_1 parameter depends only on the antenna array topology. An optimization of a problem means searching for an optimal configuration where CRB_1 is minimal. It is clear from Eq. (13) that the minimum CRB_1 happens when the Δ is at its maximum.

The analytical formula for CRB parameter is:

$$\Delta = \frac{d_1^2 + \dots + d_M^2}{M} - \frac{(d_1 + \dots d_M)^2}{M^2}. \quad (14)$$

The Eq. (14) can be rewritten into the following form:

$$\Delta = \left[\frac{d_1^2 + \dots + d_M^2}{M} - \frac{(d_1 + \dots d_M)^2}{M^2}\right] = = E[d_m^2] - E[d_m]^2, \quad (15)$$

and consequently, into the following formula:

$$\Delta = E\left[(d_m - E(d_m))^2\right] = \frac{1}{M}\sum_{m=1}^{M}\left(d_m - \overline{d}\right)^2, \quad (16)$$

where $\overline{d}$ is the mean value of the all base antenna elements, d_m is the distance between m-th element and 1-th element of the NLA $(m = 1, \dots, M)$.

Equation (16) is then defined as the variance of all the antenna element distances. In the study described below, the number of antenna elements was low and the Eq. (14) was then used for optimization of the antenna element positions.

4. Simulation Results

4.1. 3-Element NLA Configuration Testing

This chapter describes a simulation of NLA configuration with 3 antenna elements, a constant aperture $B = 13d$ and a fixed signal source DOA $\theta_0 = -20°$. In order to compare the results of the individual configurations, a calculation of Root Mean Square Error (RMSE) from the following formula was used:

$$\mathrm{RMSE} = \sqrt{\frac{1}{P}\sum_{i=1}^{P}(\theta - \theta_0)^2}, \quad (17)$$

where θ is the estimated incoming signal source, θ_0 is the desired fixed signal's DOA and P is the number of the computing trials.

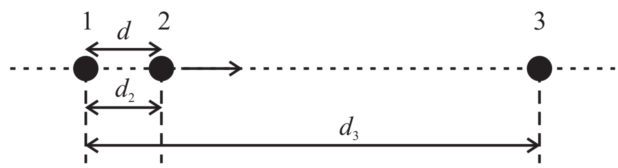

Fig. 3: 3-element NLA configuration.

The NLA will be enlarged to maximum aperture and its configuration will then be $d_{NLA} = [0\ d_2\ 13]d$, where d_2 will be changing in the interval $\langle 1, 12\rangle$ with $d = \lambda/2$. Simulation parameters are summarized in Tab. 1.

It is apparent from Fig. 4 that the NLA configuration does not affect the resulting signal's DOA for high

Tab. 1: Basic simulation parameters.

	Parameter	Value
Basic NLA parameter	Number of antenna elements (-)	3
	Sector (°)	−90 to 90
	Carrier frequency (MHz)	2450
	NLA configuration	$d_{NLA} = [0\ d_2\ 13]d$
	Number of signal samples	200
	Number of trials (-)	1000
1 signal source	Desired DOA (°)	−20
	Signal-to-Noise Ratio (SNR) (dB)	−10 to 5

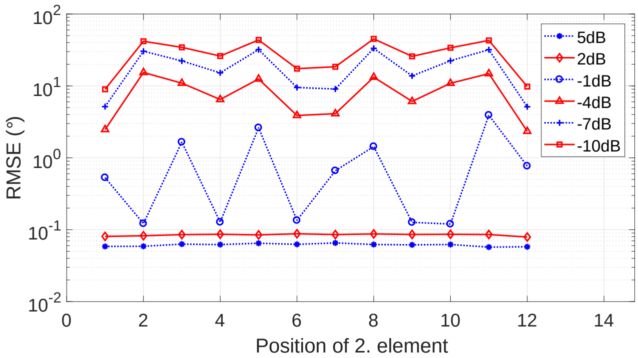

Fig. 4: The dependency of RMSE on 2nd antenna element position (SNR = −10 dB to 5 dB).

SNR (SNR > 2 dB). For SNR < 2 dB, the estimated DOAs are highly variable and depend on the second NLA antenna element position.

Figure 5 shows a histogram of an evaluated signal's DOA where appropriate and false DOAs are visible. This DOA ambiguity is related to low SNR that worsens the RMSE parameter (Fig. 4).

Next, Fig. 6 presents simulation outputs for very low SNR values in the interval (−10 dB, −5 dB). This simulation shows that for the second antenna proximity, close to the first and the third antenna element, the RMSE is better than in other positions (red ellipses). RMSE values for near to first or third antenna element

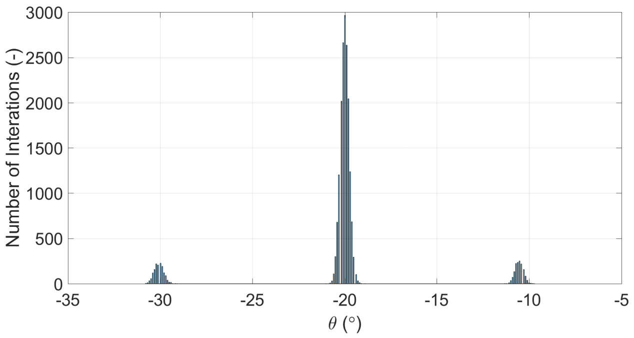

Fig. 5: Histogram of evaluated DOA for $d_2 = 1$, SNR = −7 dB, number of trials = 20000.

position correspond to the expected values mentioned in the analytical theory part of the previous chapter.

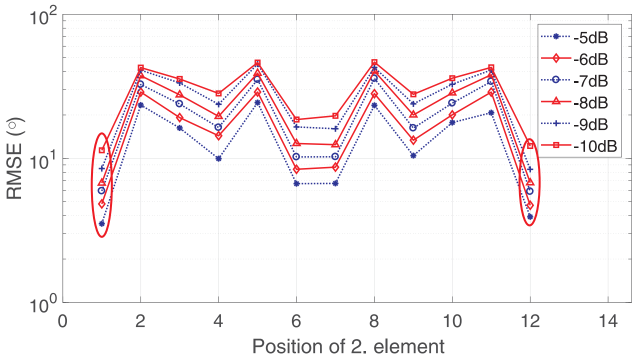

Fig. 6: The relation of RSME to the position of the second antenna array element (SNR = −10 dB to −5 dB).

4.2. 4-Element NLA Configuration Testing

The 4-antenna element NLA configuration was simulated for two cases. In the first case, only one incoming signal source was used, while in the second case, two incoming signal sources were used. Five different antenna array configurations with fixed number of antenna elements and different apertures were chosen for the simulations.

The NLA configurations were chosen according to Eq. (11). The first configuration is a standard ULA with 4 antenna elements. The second configuration (NLA d1) is an NLA as Minimum Redundant Array (MRA), which could establish a virtual array with aperture equal to a 7-element ULA. The third configuration (NLA d2) is a random NLA with an aperture of $B = 10 \cdot d$. The fourth configuration (NLA d3) is projected according to [12] as an optimal configuration for phase interferometric implementation. The fourth configuration guaranties the parallelity of phase spaces with equal distances. The fifth configuration (NLA dCRB) is an optimal according to the CRB (Eq. (13)), which has the same aperture as fourth configuration.

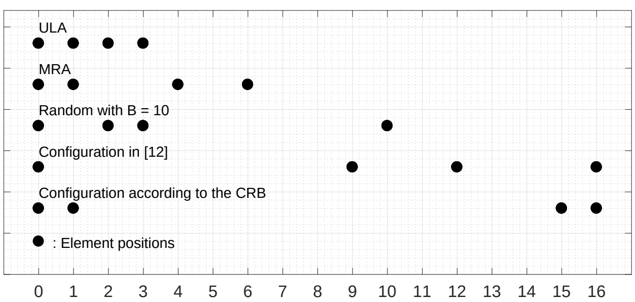

Fig. 7: Five selected antenna array configurations.

Figure 8 shows the advantage of using NLA in comparison to ULA. For the accuracy evaluation of various antenna arrays, a Monte Carlo method with 1000 trials was used. The RMSE values for individual NLA configurations are shown in Fig. 9.

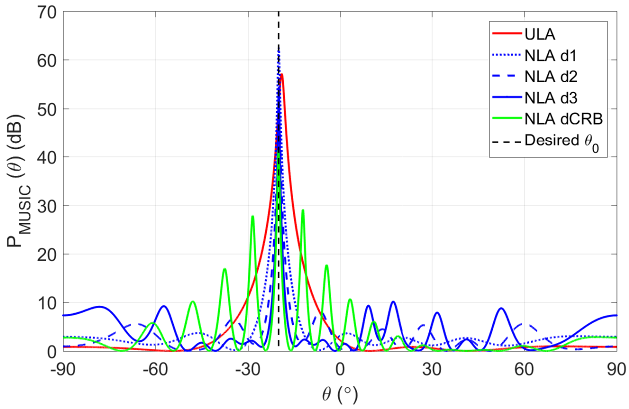

Fig. 8: MUSIC spectra with ULA and NLA (SNR = −7 dB).

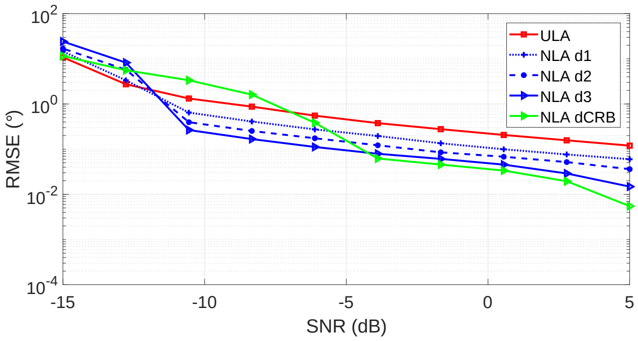

Fig. 9: RMSE dependency on SNR for individual NLA configurations.

The computed values of RMSE in Fig. 9 show that four NLA configurations reached better results for SNR higher than −6 dB. In the opposite case, (with low SNR) the RMSE worsens when using the NLA until DOA information in a given area is completely lost.

The angle resolution evaluation was provided in next step, with two uncorrelated incoming signal sources being used at DOAs of $\theta_1 = -20°$ and $\theta_2 = -30°$.

The MUSIC spectrum in Fig. 10 shows that in an NLA configuration, the method can visibly distinguish between the two uncorrelated signals arriving in the direction of −30° and −20° at SNR = 0 dB.

The probability of resolution of two incoming signals was tested for all defined configurations. The MUSIC method can resolve two targets of closed angles with 100 % certainty for the probability of resolution equals to 1. The development of probability of resolution was evaluated for various SNRs and a constant number of trials as seen in Fig. 11.

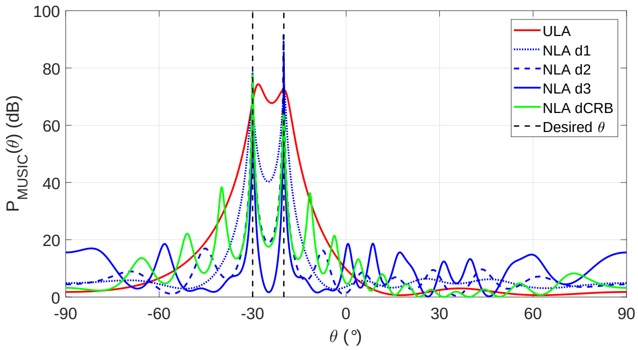

Fig. 10: MUSIC spectra for two uncorrelated incoming signals.

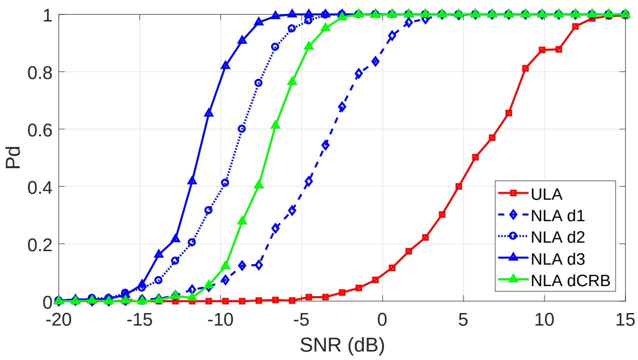

Fig. 11: Probability of resolution of two uncorrelated incoming signals.

For simulations, two incoming signal sources with DOAs −5° and 5° were used. Figure 11 shows that, in an NLA configuration, two signals can be distinguished better than in an ULA configuration, even with lower SNR.

5. Practical Test Results

Practical tests were carried out using a 4-antenna element NLA configuration in an anechoic chamber. For using one transmitter, testing equipment with a four-channel oscilloscope connected to a PC was assembled. The testing equipment block diagram is shown in Fig. 12.

The transmitter generates a CW signal with a frequency of 2450 MHz and is positioned at a distance of $L = 5.2$ m from the receiving antenna array.

The rectangular patch antennas [13] were designed and tuned to 2450 MHz. The particular designed antenna is fabricated on the PTFE/Teflon substrate with a relative permittivity of 2.1, and the loss tangent of 0.001 at 2450 MHz. Its dimensions are shown in Fig. 13.

The measured return loss of the antenna element is shown in Fig. 14. The figure shows that the −10 dB return loss bandwidth of the antenna is 2400–2500 MHz,

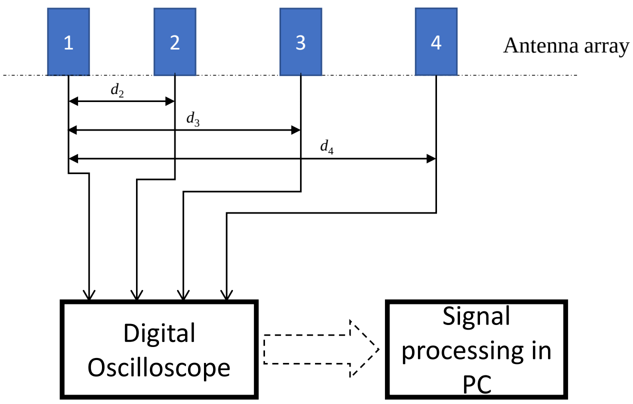

Fig. 12: Testing equipment block diagram of 4-elements configuration.

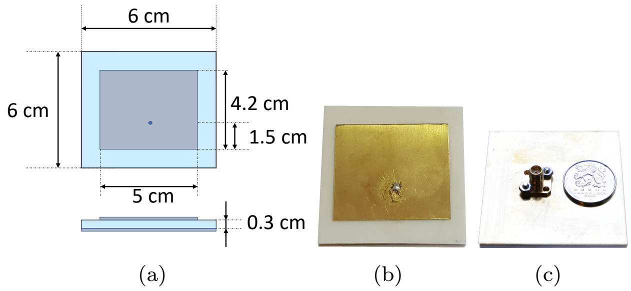

Fig. 13: (a) Geometry of a patch antenna (b) top view (c) bottom view of designed patch antenna.

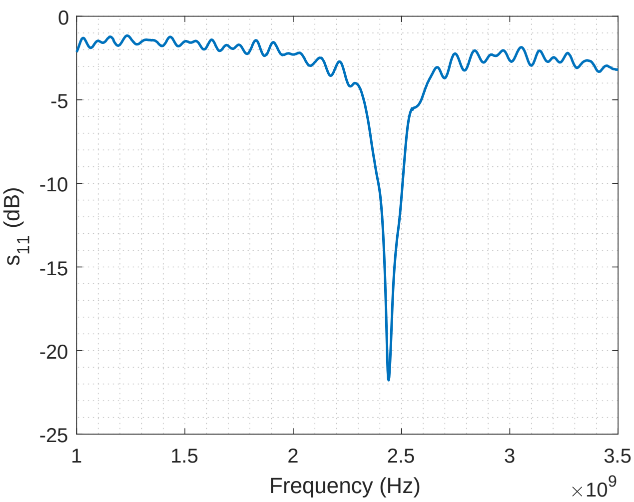

Fig. 14: Measured return loss (s11) for the designed patch antenna.

thus tuned frequency is safely located within the −10 dB bandwidth of the antenna. The radiation pattern in horizontal plane is plotted in Fig. 15, the half-power beamwidth in the horizontal plane is 80°.

The reference distance between the antenna elements is set to $d = 0.061$ m ($\approx \lambda/2$). Four antenna elements were distributed across the line in an NLA configuration $d_{NLA} = [0\ 1\ 4\ 6]d$. The practical tested an-

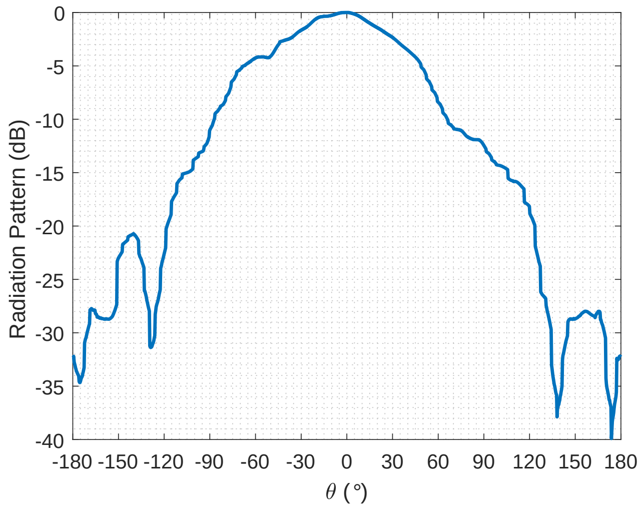

Fig. 15: Measured radiation pattern of the antenna element.

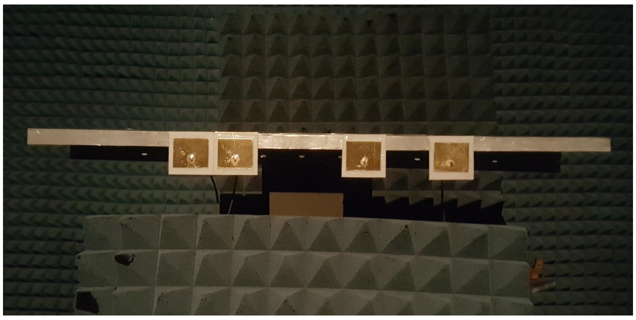

Fig. 16: Photograph of tested NLA configuration in anechoic chamber.

tenna array applied during measurement in anechoic chamber is shown in Fig. 16. Signals from the individual antenna elements were connected to inputs of a 4-channel R&S RTO 1014 oscilloscope via coaxial cables with constant lengths.

The MUSIC spectrum of the measured signals for one transmitter is shown in Fig. 17. The main lobe of the computed MUSIC spectra is clearly visible, and its width is narrower than the widths of lower side lobes as seen in Fig. 17. This MUSIC spectra, computed from measured signals using NLA, verified the applicability of the MUSIC method.

Tab. 2: Measurement results for different DOA signal.

Actual DOA signal	$-21°$	$-11.5°$	$11°$	$23°$
Average of estimated DOA	$-20.2°$	$-11.05°$	$11.9°$	$23.65°$
Variance δ^2	0.0139	0.0129	0.013	0.0134

In this case, the position of transmitter was located at four different angles for testing DOA signal. Each DOA signal was measured by 25 measuring cycles, the mean estimated DOA and variance DOA are presented in Tab. 2. Even though the measured DOA is different

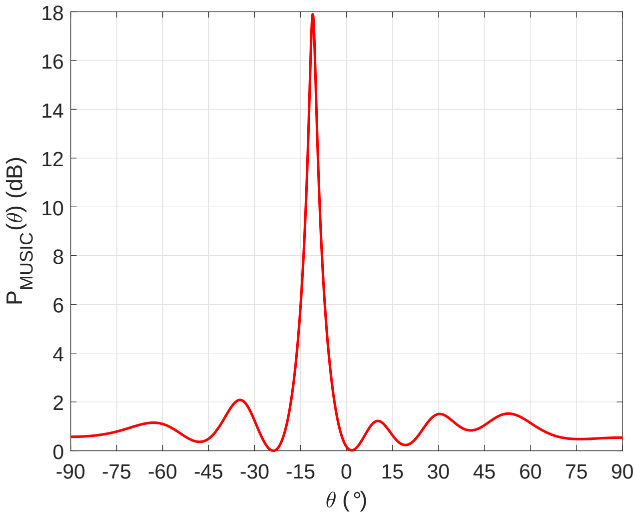

Fig. 17: MUSIC spectrum of signal measured using the NLA configuration $d_{NLA} = [0\ 1\ 4\ 6]d$, $(\theta = -11.04°)$.

from the expected, the variance δ^2 is acceptably low. This means the error is not principally a methodical error but may be caused by different cable lengths or by transmitter to NLA distances.

6. Conclusion

By verifying the effects of a nonlinear antenna array configuration on the results of signal DOA estimation, the theoretical hypotheses were confirmed. A higher probability of resolution and accuracy of the signal's DOA was achieved by using an antenna array of a larger aperture. The basic options for antenna element position optimization were discussed from the viewpoints of both analytical CRB solutions and the simulation results. The results confirmed that the accuracy in distinguishing and determining the signal DOA is dependent on the antenna array configuration, especially the antenna array aperture and the SNR parameter. Furthermore, the configuration in [12] could be considered as an optimal NLA configuration for determining the one or two incoming signals. The practical tests showed that the NLA configuration can be used to determine signal DOA for a very good result. A future paper will focus on optimizing the configuration of a 2-D antenna array extending the assessment to include a determination of the elevation angle direction of incoming signals.

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