Position Estimation Bias Analysis of a Multilateration System with a Reference Station Selection Technique

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DOI: 10.15598/aeee.v16i3.2776

Abstract. *Multilateration (MLAT) system estimates the position of an aircraft using Time Difference of Arrival (TDOA) measurements estimated at spatially located Ground Receiving Station (GRS) pairs with a lateration algorithm. The Position Estimation (PE) accuracy of an MLAT system depends on several factors, one of which is the choice of reference station used to generate the TDOA estimations for use with the lateration algorithm. Furthermore, the closed-form lateration algorithm is known to introduce bias in the PE process. Thus, a bias analysis and improvement in the PE accuracy of an MLAT system with a reference selection technique is presented in the paper. The analysis is carried out for a square GRS configuration with each GRS equipped with a receiver whose Time of Reception (TOR) error Standard Deviation (SD) is assumed 1 nsec. Monte Carlo simulation result of lateration algorithm with reference selection technique shows a reduction of at least 75 % in both the overall PE Mean Square Error (MSE) and bias. Furthermore, the PE Root Mean Square Error (RMSE) obtained by the lateration algorithm is reduced by at least 50 % out of which 36 % of the reduced PE RMSE is contributed only by the TOR estimation error.*

Keywords

Bias analysis, MLAT, PE error, reference selection, TDOA.

1. Introduction

The Position Estimation (PE) process of a Multilateration (MLAT) system is a two-stage process [1]: (1) Time Difference of Arrival (TDOA) estimation and (2) using the TDOA estimates to derive the aircraft position with a lateration algorithm. Several approaches to TDOA. estimation have been presented [2], [3], [4] and [5] and can be grouped as direct and indirect approaches. The direct approach is mostly used in a non-cooperative environment, such as in Electronic Warfare (EW) by the military in which no knowledge of the received signal characteristic is known [2] and [6]. The TDOA measurement is estimated directly from the signal detected at each spatially located Ground Receiving Station (GRS) using techniques such as cross-correlation and generalized cross-correlation [6]. With known knowledge of the signal characteristics, such as the preamble in an automatic surveillance dependent broadcast (ADS-B) packet, the indirect approach is used [3] and [5]. It involves estimating of the Time of Reception (TOR) of the aircraft transponder emission (Mode A, Mode C and Mode S replies) detected at each of the deployed GRSs. The TORs are subsequently sent to the Central Processing Station (CPS) where a pairwise subtraction operation of the TORs is performed to obtain the TDOA measurements. Several techniques to estimate the TOR of the received signal have been reported [5] and [7], but the leading-edge pulse detection technique is the most commonly used for the Mode A, Mode C, Mode S and ADS-B aircraft transponder replies [7]. It involves getting the time instance that corresponds to the overcoming of an assigned threshold by the first leading pulse edge of the signal through a clock which each GRS is equipped with. In this paper, the indirect approach to TDOA estimation is adopted as a technique used in the civil aviation.

The TDOA measurements estimated in the first stage are used with a lateration algorithm to estimate the aircraft position in the second stage [1] and [2]. A nonlinear relationship exists between the input (TDOA measurement) and the output variable (aircraft position) of the lateration algorithm [1]. Several approaches to linearize this relationship have been proposed [1], [8], [9], [10], [11], [12], [13] and [14] which resulted into the different types of lateration algorithms, namely closed-form and open-form [1] and [14]. The open-form lateration algorithm utilizes linear approximation techniques such as the Gauss–Newton algorithm to obtain the linear relationship and uses iteration process while minimizing a maximum likelihood cost function to obtain the aircraft position [1] and [11]. Due to the iteration process, convergence is an issue and therefore not suitable for real-time implementation [13]. The closed-form approach involves the use of algebraic manipulation to obtain the linear relationship between the two variables which results in a set of plane equations with the aircraft position as unknown [15] and [16]. Because no iteration process is involved, convergence is not an issue and is most suitable for a passive real-time surveillance system. In this paper, the closed-form lateration algorithm developed in [9] is considered as commonly used for passive MLAT system. The closed-form lateration algorithm is known for high PE bias resulting to high PE error [10]. This is due to the algebraic manipulation of the TDOA measurement equations used to obtain the linear relationship between the two variables. Several techniques have been proposed to reduce the high PE error characteristics of the lateration algorithm [9], [10], [17] and [18]. A Total Least Square (TLS) based method is proposed to reduce the bias of the lateration algorithm thus reducing the overall PE error of the system [10]. Another technique is based on GRS reference selection suitable to generate the TDOAs used with the lateration algorithm [9], [17] and [18]. A GRS reference selection technique based on condition number calculation of a matrix with TDOA measurements as it only entries is proposed in [9]. It has been shown that the technique improved the PE accuracy of the lateration algorithm. However, no detailed analysis in terms of bias analysis is presented to determine how the reference selection technique improved the PE accuracy. Thus, in this research, the PE performance and bias analysis of the MLAT system with and without the reference station selection technique is presented. This leads to determine if there is any reduction in the bias introduced by the lateration algorithm as well as the improvement in the PE accuracy of the MLAT system with the reference selection technique.

The reminder of the paper is organized as follows. Section 2. presents the methodology of the direct approach to TDOA estimation. In Sec. 3. , a brief description of the MLAT aircraft PE methodology as previously reported in [9] is presented while in Sec. 4. , the approach to determine the bias introduced by the lateration algorithm in the PE process is presented. This is followed by simulation result and discussion in Sec. 5. and finally, the conclusion in Sec. 6.

2. Indirect Approach to TDOA Estimation

In this section of the paper, the methodology of generating the TDOA estimation set based on the indirect approach is presented. Let T^i_{rep} and T^m_{rep} be the actual TOR of the signal at the i-th and m-th GRS pair. Mathematically, the TDOA of the signal between the i-th and m-th GRS pair is obtained from the TORs as follows:

$$\tau_{im} = T^i_{rep} - T^m_{rep}. \tag{1}$$

In practical application, the accuracy at which the TOR is obtained depends on several factors which are synchronization error between GRSs, quantization error, and time-stamp resolution of the receiver used in [2], [3] and [5]. In this paper, the TOR error is assumed to include all these factors. Modelling the TOR estimation error as zero mean Gaussian random variable with probability density function as $N(0,\sigma)$ [14], the measured TORs of the signal at the i-th and m-th GRS respectively are:

$$\hat{T}^i_{rep} = T^i_{rep} + N\left(0,\sigma\right), \tag{2}$$

$$\hat{T}^m_{rep} = T^m_{rep} + N\left(0,\sigma\right), \tag{3}$$

where σ in Eq. (2) and Eq. (3) is the TOR error Standard Deviation (SD) which ranges from 1 to 20 nsec [2].

The measured TDOA after substituting Eq. (2) and Eq. (3) is obtained as:

$$\hat{\tau}_{im} = \hat{T}^i_{rep} - \hat{T}^m_{rep}. \tag{4}$$

Depending on the number of GRSs deployed and number for stations used as reference to generate the TDOA estimations, several TDOA measurements in the form of Eq. (4) are obtained [14]. Let there be four GRSs deployed each labelled i-th, j-th, k-th and

m-th while a pair of GRS is chosen as reference for the TDOA estimation. With the i-th and j-th GRSs chosen as the reference pair and the k-th and m-th as the non-reference GRSs, a TDOA measurement set is obtained consisting of four TDOA measurements as presented in [9] and are expressed as follows:

$$\hat{\tau}_{im} = \hat{T}^{i}_{rep} - \hat{T}^{m}_{rep}, \tag{5}$$

$$\hat{\tau}_{jm} = \hat{T}^{j}_{rep} - \hat{T}^{m}_{rep}, \tag{6}$$

$$\hat{\tau}_{ik} = \hat{T}^{i}_{rep} - \hat{T}^{k}_{rep}, \tag{7}$$

$$\hat{\tau}_{jk} = \hat{T}^{j}_{rep} - \hat{T}^{k}_{rep}. \tag{8}$$

The TDOA measurements in Eq. (5) to Eq. (8) are subsequently used by the lateration algorithm to estimate the position of the aircraft. This is presented in the next section.

3. MLAT Position Estimation Methodology

A brief description of the close-form lateration algorithm for aircraft PE by the MLAT system is first presented in this section. This is followed by a summary of the reference station selection technique based on condition number calculation as proposed in [9] to generate the suitable TDOA measurement set for use with the lateration algorithm.

3.1. MLAT Closed-Form Lateration Algorithm

Let the coordinates of the aircraft with TDOA measurements obtained in Eq. (5) to Eq. (8) be $\mathbf{x} = (x, y, z)$ and the coordinates of the i-th, j-th, k-th and m-th GRSs be $\mathbf{S}_i = (x_i, z_i, z_i)$, $\mathbf{S}_j = (x_j, z_j, z_j)$, $\mathbf{S}_k = (x_k, z_k, z_k)$ and $\mathbf{S}_m = (x_m, z_m, z_m)$ respectively. The TDOA measurements in Eq. (5) to Eq. (8) respectively relates to the aircraft position as follows:

$$\hat{d}_{im} = c \cdot \hat{\tau}_{im} = \|\mathbf{x} - \mathbf{s}_i\| - \|\mathbf{x} - \mathbf{s}_m\|, \tag{9}$$

$$\hat{d}_{jm} = c \cdot \hat{\tau}_{jm} = \|\mathbf{x} - \mathbf{s}_j\| - \|\mathbf{x} - \mathbf{s}_m\|, \tag{10}$$

$$\hat{d}_{ik} = c \cdot \hat{\tau}_{ik} = \|\mathbf{x} - \mathbf{s}_i\| - \|\mathbf{x} - \mathbf{s}_k\|, \tag{11}$$

$$\hat{d}_{jk} = c \cdot \hat{\tau}_{jk} = \|\mathbf{x} - \mathbf{s}_j\| - \|\mathbf{x} - \mathbf{s}_k\|, \tag{12}$$

where $c = 3 \cdot 10^8$ m·s^{-1}, $\hat{d}_{im}$ and $\hat{d}_{jm}$ are the TDOA measurements obtained using the m-th non-reference GRS with the i-th and j-th GRS as references respectively while $\hat{d}_{ik}$ and $\hat{d}_{jk}$ are the TDOA measurements obtained using the k-th non-reference GRS with the i-th and j-th GRS as references respectively. The $\|.\|$ denotes the 2-norm operator.

Algebraically manipulating Eq. (9) to Eq. (12) as done in [8] and [9] results in a pair of 3-D plane equation in the form:

$$A_{ikm} = xB_{ikm} + yC_{ikm} + zD_{ikm}, \tag{13}$$

$$A_{jkm} = xB_{jkm} + yC_{jkm} + zD_{jkm}, \tag{14}$$

where the coefficients of Eq. (13) and Eq. (14) are functions of the TDOA measurements and GRS coordinate which are found in [9]. The matrix form of Eq. (13) and Eq. (14) is defined as follows:

$$\begin{bmatrix} B_{ikm} & C_{ikm} & D_{ikm} \\ B_{jkm} & C_{jkm} & D_{jkm} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} A_{ikm} \\ A_{jkm} \end{bmatrix}, \tag{15}$$

$$\mathbf{H}_{ij} \cdot \mathbf{x}_e = \mathbf{a}_{ij}. \tag{16}$$

The Eq. (16) is an underdetermined least square equation and is the PE mathematical equation for the 3-D minimum configuration MLAT system. Detailed derivation of finding the aircraft position using Eq. (16) is presented in [8] and [9] and is used in this research for the aircraft PE.

3.2. Selection of Reference Station to Generate TDOA Measurements for the Lateration Algorithm

The choice of reference station used to generate the TDOA measurements in Eq. (5) to Eq. (8) and subsequently the plane equations presented in Eq. (13) and Eq. (14) contribute to the overall PE accuracy of the MLAT system [9] and [18]. Thus, the choice of reference station is critical in ensuring accurate aircraft positions which are obtained. In this paper, the reference selection technique proposed in [9] is used in choosing the suitable GRS reference station pair to generate the TDOA measurements in Eq. (5) to Eq. (8). With the deployed ground stations labelled GRS-1, GRS-2, GRS-3 and GRS-4, Tab. 1 shows all the possible combinations of reference GRS pairs.

Summary of the approach to select the suitable GRS reference station pair shown in Tab. 1 to generate the TDOA measurements for use with the lateration algorithm is illustrated as follows [9]:

- **Step-1:** Using the measured TORs of the aircraft emission at each GRS, obtain the TDOA measurements set in the form of Eq. (17) using all the possible GRS reference pair combinations shown in Tab. 1.

$$\mathbf{T}_{ijmn} = [\hat{\tau}_{ik}, \hat{\tau}_{im}, \hat{\tau}_{jk}, \hat{\tau}_{jm}] . \qquad (17)$$

- **Step-2:** Using the TDOA measurement set obtained in step-1 for each GRS pair, substitute into Eq. (18) and solve for χ.

$$\chi = \left(\frac{\hat{\tau}_{jm} \cdot \hat{\tau}_{jk}}{\hat{\tau}_{im} \cdot \hat{\tau}_{ik}}\right) + \left(\frac{\hat{\tau}_{im} \cdot \hat{\tau}_{ik}}{\hat{\tau}_{jm} \cdot \hat{\tau}_{jk}}\right) . \qquad (18)$$

- **Step-3:** Choose the GRS pair with the least χ value from step-2 as the reference pair to be used to generate the TDOA measurements in Eq. (5) to Eq. (8) and subsequently the plane equations (Eq. (13) and Eq. (14)) for the PE.

- **Step-4:** Estimate the position of the aircraft using the lateration algorithm presented in Sec. 3.1.

Tab. 1: Possible pair combinations of GRSs.

GRS Reference pair	i-th	j-th
Pair 1	GRS-1	GRS-2
Pair 2	GRS-1	GRS-3
Pair 3	GRS-1	GRS-4
Pair 4	GRS-2	GRS-3
Pair 5	GRS-2	GRS-4
Pair 6	GRS-3	GRS-4

4. Closed-Form Lateration Algorithm Bias Estimation

The lateration algorithm presented in Sec. 3.1. is known to introduce bias in the PE process. The bias introduced contributes to the overall PE Mean Square Error (MSE). Mathematically, the overall PE MSE based on N realization Monte Carlo simulation is expressed as [19]:

$$PE_{MSE} = \\ = \frac{1}{N} \cdot \sum_{n=1}^{N} \left[(\hat{x}_n - x)^2 + (\hat{y}_n - y)^2 + (\hat{z}_n - z)^2 \right] , \qquad (19)$$

where (x, y, z) is the known aircraft position and $(\hat{x}_n, \hat{y}_n, \hat{z}_n)$ is the estimated aircraft position at the n-th Monte Carlo simulation realization.

The bias squared in the PE error introduced by the lateration algorithm based on N realizations of Monte Carlo simulation is mathematically expressed as [19]:

$$\gamma_{PE}^2 = \frac{1}{N} \cdot \left(\begin{array}{l} \left\| \sum_{n=1}^{N} (\hat{x}_n - x) \right\|^2 + \left\| \sum_{n=1}^{N} (\hat{y}_n - y) \right\|^2 \\ + \left\| \sum_{n=1}^{N} (\hat{z}_n - z) \right\|^2 \end{array} \right) . \qquad (20)$$

The PE MSE in Eq. (19) is related to the bias squared in Eq. (20) as follows [20]:

$$PE_{MSE} = \gamma_{PE}^2 + \sigma_{PE}^2 , \qquad (21)$$

where σ_{PE}^2 is the variance in the PE error due to TOR error SD.

Using Eq. (21), the amount of bias introduced by the lateration algorithm and the actual PE error due to the TOR error are determined.

5. Simulation Result and Discussion

In this section of the paper, the bias introduced by the lateration algorithm in Sec. 3.1. and the PE error comparison are determined for both the lateration algorithms with the reference selection technique and with the fixed reference pair technique as previously done in [15]. This is to determine the percentage contribution of the bias introduced by the lateration algorithm in the overall PE MSE as well as the actual amount of PE error that is due to the TOR estimation error. For the lateration algorithm with the fixed reference pair technique, the GRS-1 and GRS-2 are used as the reference pair stations for TDOA measurement set generation.

The TOR estimation is not within the scope of this study. However, it is assumed that each of the GRSs is equipped with a receiver with a TOR error SD of 1 nsec (0.3 m) based on the result presented in [2] and [21]. The configuration in which the GRSs are deployed also contributes to the PE accuracy of the MLAT system [22] and [23]. It is suggested in [23] that for a total of four GRSs, the square configuration resulted in the best PE accuracy and is adopted in this paper. The distribution of the GRSs is shown in Fig. 1.

The PE MSE and bias squared based on Eq. (19) and Eq. (20) respectively are determined for both lateration algorithms, with and without the reference selection technique. The results are obtained at ten randomly selected aircraft positions with coordinate as presented in Tab. 2 based on $N = 200$ realizations of Monte Carlo simulation and are presented in Tab. 3.

From Tab. 3, both the PE MSE and PE bias squared of the lateration algorithm with the reference selection

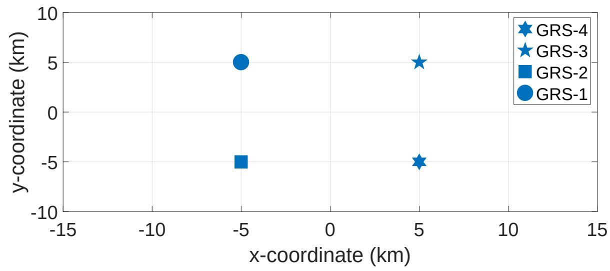


Fig. 1: Distribution of the GRSs in a Square configuration with 10 km separation.

Tab. 2: Selected aircraft positions of bias analysis.

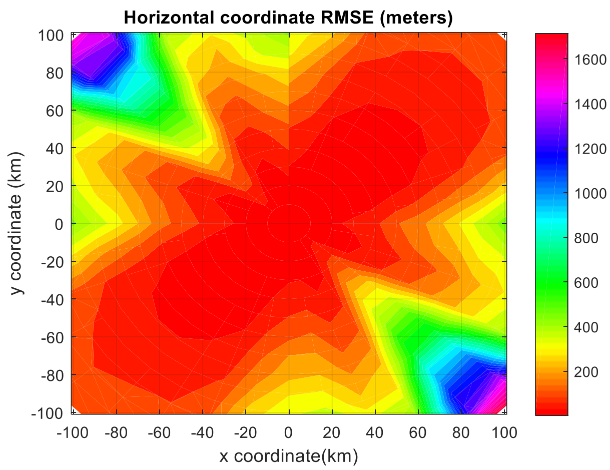
Aircraft Location	Aircraft Coordinate		
	x (km)	y (km)	z (km)
A	5	20	2
B	20	35	
C	42	42	
D	69	20	
E	97	26	
F	10	3	2
G	26	15	
H	35	35	
I	35	61	
J	13	13	

technique are less than that obtained with the fixed reference pair technique. At aircraft position A, the PE MSE of the lateration algorithm with the reference selection technique and using the fixed reference pair technique are $1.10 \cdot 10^{-4}$ km^2 and $1.30 \cdot 10^{-4}$ km^2 respectively, while the PE bias squared are $0.68 \cdot 10^{-4}$ km^2 and $0.82 \cdot 10^{-4}$ km^2 respectively. The absolute difference in the PE MSE and the PE bias squared are $0.2 \cdot 10^{-4}$ km^2 and $0.14 \cdot 10^{-4}$ km^2 respectively. Extending the analysis to aircraft at positions B, C, D, E, F, G, H, I and J, the absolute differences in the PE MSE are $17.06 \cdot 10^{-4}$ km^2, $169.31 \cdot 10^{-4}$ km^2, $1100.93 \cdot 10^{-4}$ km^2, $13030 \cdot 10^{-4}$ km^2, $1.32 \cdot 10^{-4}$ km^2, $4.46 \cdot 10^{-4}$ km^2, $16.84 \cdot 10^{-4}$ km^2, $46.07 \cdot 10^{-4}$ km^2, and $13.34 \cdot 10^{-4}$ km^2 respectively while the PE bias squared are $11.2 \cdot 10^{-4}$ km^2, $106.15 \cdot 10^{-4}$ km^2, $688.31 \cdot 10^{-4}$ km^2, $7975.00 \cdot 10^{-4}$ km^2, $0.86 \cdot 10^{-4}$ km^2, $2.83 \cdot 10^{-4}$ km^2, $10.63 \cdot 10^{-4}$ km^2, $28.79 \cdot 10^{-4}$ km^2, and $7.94 \cdot 10^{-4}$ km^2 respectively.

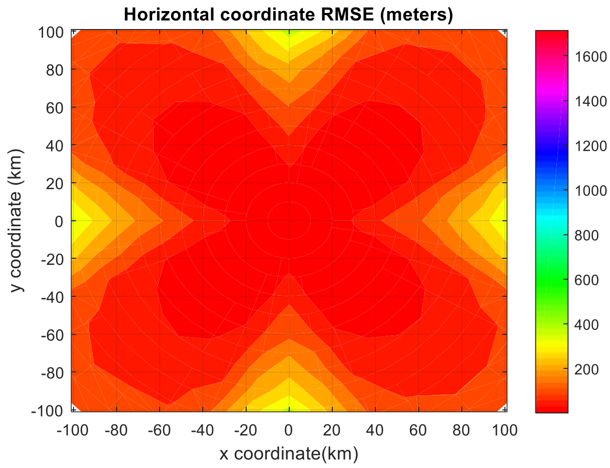
On the average, based on the selected aircraft positions and square GRS configuration, the reference selection technique reduced both the PE MSE and bias squared of the lateration algorithm by at least 75 % compared to using the fixed reference pair technique. However, for both technique, about 64 % of the overall PE MSE is due to bias. This means that the reference selection technique reduced the overall PE MSE of the lateration algorithm by at least 75 % but the percent contribution of the bias square in the reduced PE MSE value is about 64 % for both techniques. Thus, with or without the reference selection technique, the TOR error contributed only 36 % to the overall PE MSE despite a reduction of about 75 % in the PE MSE by the reference selection technique.

Tab. 3: PE error variance of the lateration algorithm with and without the reference selection technique.

Aircraft location	Without reference selection technique (10^{-4})		With reference selection technique (10^{-4})	
	PE_{MSE} (km^2)	γ_{PE} (km^2)	PE_{MSE} (km^2)	γ_{PE} (km^2)
A	1.30	0.82	1.10	0.68
B	22.00	14.33	4.94	3.13
C	191.50	120.40	22.19	14.25
D	1175	736.10	74.07	47.79
E	13370.00	8195.00	340.00	220.00
F	1.35	0.88	0.03	0.02
G	5.08	3.24	0.62	0.41
H	20.35	12.83	3.50	2.20
I	64.44	40.22	18.37	11.43
J	26.81	16.64	13.47	08.70



(a) Fixed reference pair (GRS-1 and GRS-2 as reference pair).



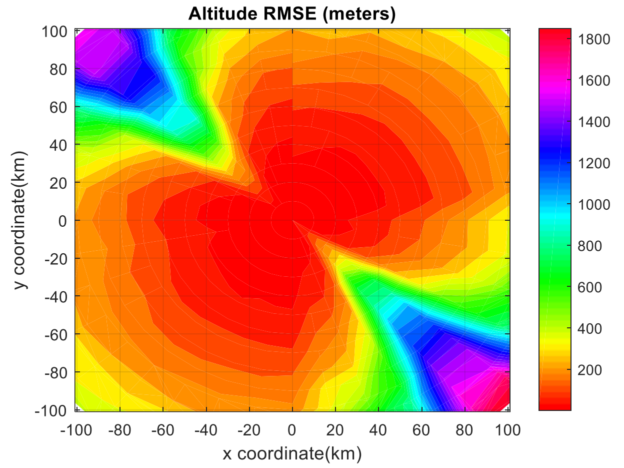
(b) Horizontal coordinate RMSE comparison within 100 km system coverage.

Fig. 2: Horizontal coordinate RMSE comparison within 100 km system coverage.

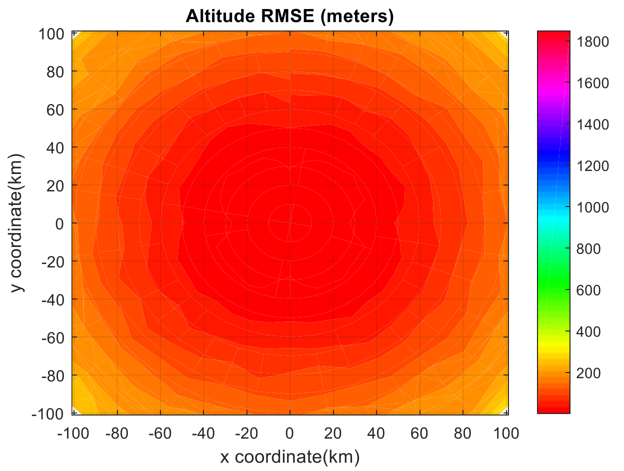
The Fig. 2 and Fig. 3 respectively show the horizontal coordinate and altitude RMSEs within a system coverage of about 100 km. The mathematical expressions of the horizontal coordinate and altitude RMSE based on N realization Monte Carlo simulation are respectively expressed as follows:

$$H_{rmse} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left[(\hat{x}_n - x)^2 + (\hat{y}_n - y)^2\right]}, \tag{22}$$

$$Z_{rmse} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\hat{z}_n - z)^2}. \tag{23}$$



(a) Fixed reference pair (GRS-1 and GRS-2 as reference pair).



(b) With reference pair technique.

Fig. 3: RMSE at altitude of 2 km.

It can be seen that the position RMSE based on Fig. 2 and Fig. 3 varies with aircraft position relative to GRS configuration. Table 4 and Tab. 5 shows the horizontal coordinate and altitude RMSE comparison of the lateration algorithm with and without the reference selection technique at the randomly selected aircraft positions. Compared with using the fixed reference pair technique, the reference selection technique reduced the horizontal coordinate and altitude RMSE of the lateration algorithm. At aircraft position A, the horizontal coordinate RMSE of the lateration algorithm with the reference selection technique and using the fixed reference pair technique are 8.28 m and 8.54 m respectively, while the altitude RMSE are 6.00 m and 8.16 m respectively. The absolute horizontal coordinate and altitude RMSE differences are 0.26 m (~3 %) and 2.16 m (~26 %) respectively.

Extending the analysis to aircraft at positions B, C, D, E, F, G, H, I and J, the percentage reductions in the horizontal coordinate RMSE by the reference selection technique are about 50 %, 66 %, 78 %, 87 %, 57 %, 66 %, 60 %, 50 % and 31 % respectively. The percentage reduction in the altitude RMSEs are 58 %, 67 %,78 %, 83 %, 50 %, 56 %, 64 %, 59 %, and 46 % respectively.

Tab. 4: Horizontal coordinate RMSE comparison at TOR error SD of 1 nsec.

Aircraft position	Horizontal coordinate RMSE (m)		Absolute horizontal coordinate RMSE difference (m)
	With reference selection	Without reference selection	
A	8.28	8.54	0.26
B	12.59	24.97	12.38
C	24.33	70.56	46.23
D	52.52	243.33	190.81
E	146.50	1213	1066.5
F	3.61	8.42	4.81
G	7.47	22.19	14.72
H	16.17	40.05	23.88
I	36.19	71.07	34.88
J	112.30	162.40	50.1

Tab. 5: Altitude RMSE comparison at TOR error SD of 1 nsec.

Aircraft position	Altitude RMSE (m)		Absolute altitude RMSE difference (m)
	With reference selection	Without reference selection	
A	6.00	8.16	2.16
B	19.08	45.68	26.6
C	46.02	140.50	94.48
D	87.89	414.40	326.51
E	145.60	842.50	696.9
F	3.65	7.39	3.74
G	4.00	9.03	5.03
H	10.95	30.21	19.26
I	22.31	54.4	32.09
J	37.93	69.97	32.04

On the average based on the selected aircraft positions and the square GRS configuration, the reference selection technique reduced both the horizontal coordinate and altitude RMSEs of the lateration algorithm by at least 50 % compared to that obtained with the fixed GRS reference pair technique. Based on the bias analysis presented earlier, only 36 % of the horizontal coordinate and altitude RMSEs as presented in Tab. 4 and Tab. 5 respectively are due to the TOR error. Thus, the reference selection technique reduced the position RMSE of the lateration algorithm by at least 50 %, however, 36 % out of the reduced position RMSE is due to TOR error, while 64 % is caused by the lateration algorithm bias.

6. Conclusion

In this study, the improvement in the PE accuracy and the bias analysis of the closed-form lateration algorithm for a 3-D minimum configuration MLAT system with a reference selection technique is presented. The analysis is carried out at some randomly selected aircraft positions with the GRSs deployed in a square configuration and each of the GRSs is equipped with a receiver whose TOR error SD is assumed to be 1 nsec. PE Bias analysis comparison of the lateration algorithm with and without the reference selection technique based on 200 realizations of Monte Carlo simulation shows a 75 % reduction in the bias of the lateration algorithm. With or without the reference selection technique, the contribution of the lateration algorithm bias in the overall PE MSE is about 64 %. The comparison of position RMSE that is the horizontal coordinate and altitude RMSE shows a reduction of at least 50 % with the reference selection technique but only 36 % of the reduced position RMSE is caused by the TOR error. In summary, the reference selection technique reduced the overall position RMSE but not the percentage of the bias introduced by the lateration algorithm. It is possible to further reduce the position RMSE using a bias reduction technique as proposed in several articles which is not the scope of this study.

Acknowledgment

The authors thank Universiti Teknologi Malaysia (UTM) for providing the resources and support for this research.

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