Disturbances Elimination with Fuzzy Sliding Mode Control for Mobile Robot Trajectory Tracking

Walid BENAZIZA[1], *Noureddine SLIMANE*[2], *Ali MALLEM*[1]

[1]Electronics Department, Faculty of Technology, University of Batna 2,
Route de Constantine 53, Fesdis, Batna 05078, Algeria
[2]Advanced Electronics Laboratory, Faculty of Technology, University of Batna 2,
Route de Constantine 53, Fesdis, Batna 05078, Algeria

walid_benaz@hotmail.com, Slimane_doudi@yahoo.fr, Ali_mallem@hotmail.fr

DOI: 10.15598/aeee.v16i3.2767

Abstract. *The disturbances are the significant issue for the trajectory tracking of mobile robots. Therefore, an adequate control law is presented in this paper and this one is based on Global Terminal Sliding Mode (GTSM) with fuzzy control. This control law aims to guarantee the avoidance of the kinematic disturbances which are injected in the angular and linear velocities, respectively. Moreover, the dynamic model based on exponential reaching law is presented to avoid the uncertainties. The control law provides the asymptotic stability by taking into account the fuzzy rules and Lyapunov theory. Thus, the chattering phenomenon should be avoided. The simulation works prove the robustness of the proposed control law by considering the disturbances function and the robot can follow the desired trajectories.*

Keywords

Fuzzy rule, global terminal sliding mode, Lyapunov theory, mobile robot.

1. Introduction

The domain of robotic is usually known with its external disturbances and perturbations. So, the recent works are oriented on the control of this kind of systems, especially the nonholonomic systems. In this domain, it is interesting to obtain a stable movement of trajectory tracking [1] and [2]. Sliding Mode Control (SMC) is known by its solution to design the control law and the stability of disturbed systems [3] and [4]. The sliding mode depends on the sliding surface, which is exponentially stable by taking into account the Lyapunov method to guarantee the asymptotically stability of the system. A method known as conventional sliding mode suggests a discontinuous function and this one produces high frequencies known as chattering phenomenon. In this fact, many works use a higher-order sliding mode as a solution to this problem to reduce the chattering effect [5]. Furthermore, many authors have suggested methods to minimize this phenomenon by using traditional sliding mode control [6], [7] and [8]. A standard sliding mode has been exposed to be efficient control approach in the stabilization of nonlinear systems [9] and [10]. Another type of sliding mode control with observer is proposed in [11] in order to improve the efficiency of induction motor drive. The reason of using sliding mode control is in its good results and the simplicity of the control law [5] and [12].

A robust sliding mode controller for trajectory tracking for nonholonomic robot is proposed by [13], which gives a good simulation results against the uncertainty presented in the model. Another work presented in [14] proposes a new controller using sliding mode control with kalman filter for the trajectory tracking.

A fuzzy controller proposed in [15] is used to adjust the sliding surface parameters and to accelerate the system to attain the reaching phase.

The fuzzy logic is a probable solution to reduce the chattering problem as presented in [16] and [17]. Another work [18] applies a fuzzy sliding mode observer for synchronous motor, using sigmoid function, in order to minimize the effect of chattering. Many researchers suggest an adaptive fuzzy terminal sliding mode control for nonlinear systems with non-singularity in order to reach a fast convergence in presence of external disturbances [19] and [20]. In this area, to resolve the convergence states error problem in a short time, to mitigate the harmful effects of the external disturbances and to improve the robustness of mobile robot trajectory tracking, a new method is presented in this paper.

The proposed control method of trajectory tracking is divided in two subsystems, a kinematic control and a dynamic control. The kinematic control uses a Global Fast Terminal Sliding Mode control (GFTSM) in order to avoid the disturbances of the angular velocity. The main objective is to stabilize the orientation error of trajectory tracking to zero in finite time with asymptotic stability against the uncertainties and external disturbances presented in a kinematic model. The terminal attractor is implemented in the sliding surface and brings the orientation error to zero rapidly and the convergence rate to the linear sliding surface is assured. The used robot model, initially perturbed, can converge to equilibrium stable point [21] by using Global Fast Terminal Sliding Mode Control (GFTSMC). Thereafter, the work aims to provide a kinematic controller using the fuzzy logic to tackle the effect of the disturbances presented in a kinematic model and attenuate the chattering phenomenon of the linear disturbed velocity. Therefore, the parameters selection by fuzzy logic can eliminate the effect of the disturbances and tends the robot position errors to zero in short time. It has been noticed that the convergence error posture of the robot could be faster with asymptotic stability using this control law.

The dynamic control using the exponential sliding mode provides an effective method to tackle the uncertainties and disturbances presented in a dynamic model. The main advantages of this control law are the stability of the velocity error to zero for any bounded disturbances presented in the model and the guarantee of the system asymptotic stability. The performance comparison among the achieved controller in [22] and the control law, which is presented in this paper shows that GFTSMC has a perfect performance and can deal with the effect of disturbances by using the fuzzy logic. This work is organized as follows: Kinematic and Dynamic models are presented in Sec. 2. A kinematic control based on a global fast terminal sliding mode and fuzzy theory is proposed in Sec. 3. An exponential reaching law control is proposed in Sec. 4. Finally, simulation results are presented in Sec. 5.

2. Kinematic and Dynamic Models

The mobile robot used in this work is given in Fig. 1. For the robot motion, the following equations describe the movement of the robot:

$$v = R_a \left(\frac{\dot{\varphi}_r + \dot{\varphi}_l}{2} \right), \tag{1}$$

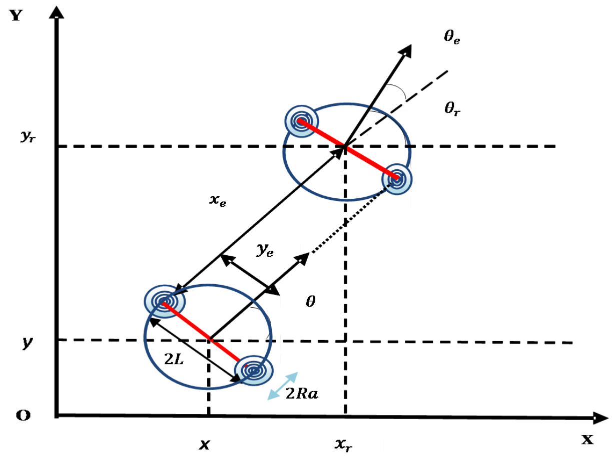

Fig. 1: Diagram of mobile robot.

$$\omega = \frac{R_a}{2L} \left(\dot{\varphi}_r - \dot{\varphi}_l \right). \tag{2}$$

$\dot{\varphi}_r$ and $\dot{\varphi}_l$ are the linear velocities of the right and left wheels, respectively. θ is the orientation angle of the mobile robot, v represents the linear velocity and R_a is the wheel radius, ω indicates the angular velocity and $2L$ is the distance separating the two wheels.

The posture of the robot is introduced with the real vector $\rho = (xy\theta)^T$ and the control vector $\gamma = (v\omega)^T$.

The disturbed kinematic model [23], [24], [25] and [26] is given by:

$$\dot{\rho} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} (\gamma + D). \tag{3}$$

D is the unknown disturbance, which is bounded [27] and expressed by Eq. (4).

$$D = [dv \quad d\omega]^T, \tag{4}$$

where $| dv | < \zeta_v$, $| d\omega | < \zeta_\omega$. dv and $d\omega$ are the disturbances of the linear and angular velocities, respectively. ζ_v and ζ_ω are positive limited constants.

The dynamic model of the mobile robot [16] and [28] is described by Eq. (5):

$$M(q)\dot{V} + V(q,\dot{q})V + F(\dot{q}) + G(q) + \tau_d = \beta(q)\tau + R(t), \tag{5}$$

where $V = (v \quad \omega)^T$ is a vector, which has as components v and ω and $\tau = (\tau_r \quad \tau_l)$ represents the torques of the right and left wheels.

$$M(q) = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \text{ and } \beta(q) = \frac{1}{R_a} \begin{bmatrix} 1 & 1 \\ L & -L \end{bmatrix}, \tag{6}$$

where m is the robot mass and I the inertia moment.

$R(t)$ represents the disturbance vector 2×1. $V(q,\dot{q})$ is the centripetal and Coriolis forces. $F(\dot{q})$ is the friction matrix.

$G(q)$ represents the gravitational vector and τ_d is an unknown disturbance.

Equation (3) and Eq. (5) of the robot model are used in order to elaborate the control law based on fuzzy global fast terminal sliding mode and exponential sliding mode control. This control algorithm is applied to satisfy the asymptotic convergence and at the same time eliminate the effect of disturbances, which are occurring in linear and angular velocities.

$$\lim_{t\to\propto} \rho_e = \lim_{t\to\propto} \parallel \rho_r(t) - \rho(t) \parallel = 0, \tag{7}$$

where: $\rho_r = (x_r \quad y_r \quad \theta_r)^T$ is the reference posture of mobile robot and $\rho_e = (x_e \quad y_e \quad \theta_e)^T$ is an error posture between the real vector ρ and the reference ρ_r.

3. Kinematic Control

Concerning the trajectory tracking, the reference posture ρ_r of the mobile robot and a desired velocity $\gamma_r = (v_r \quad \omega_r)^T$ are used.

The error posture ρ_e is represented by the following system [29]:

$$\rho_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r & -x \\ y_r & -y \\ \theta_r & -\theta \end{bmatrix}. \tag{8}$$

By introducing the nonholonomic constraints Eq. (9) into the system Eq. (8), the velocity error without disturbances is defined as in [30] by Eq. (10).

$$\dot{x}\sin\theta + \dot{y}\cos\theta = 0, \tag{9}$$

$$\dot{\rho}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} y_e\omega + v_r\cos\theta_e - v \\ -x_e\omega + v_r\sin\theta_e \\ \omega_r - \omega \end{bmatrix}. \tag{10}$$

Considering the disturbances on the velocities v and ω, Eq. (10) becomes as follows:

$$\dot{\rho}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} v_r\cos\theta_e \\ v_r\sin\theta_e \\ \omega_r \end{bmatrix} + \begin{bmatrix} -1 & y_e \\ 0 & -x_e \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v + dv \\ \omega + d\omega \end{bmatrix} =$$

$$= \begin{bmatrix} v_r\cos\theta_e - (v + dv) + y_e(\omega + d\omega) \\ v_r\sin\theta_e - x_e(\omega + d\omega) \\ \omega_r - (\omega + d\omega) \end{bmatrix}. \tag{11}$$

Assumption 1. The trajectory tracking error can be bounded and can asymptotically converge to zero when the factor $t \to \omega$, depending on the input vector $\gamma = (v \quad \omega)^T$ by taking into account the following constraints: $\mid v \mid \leq v_{\max}$ and $\mid \omega \mid \leq \omega_{\max}$.

The purpose of this control is to design a controller such as the mobile robot converges asymptotically to the desired trajectory.

3.1. Design of the Angular Velocity Control

In order to make θ_e converge to zero, the linear and the terminal sliding surface are chosen as in [31].

$$s = \dot{x} + \alpha x + \beta x^{q/p} = 0. \tag{12}$$

The Eq. (13) is written as:

$$\dot{x} = -\alpha x - \beta x^{q/p} = 0, \tag{13}$$

with α and $\beta > 0$ and $p, q (p > q)$ are positive parameters.

The benefit of the non-linear term in Eq. (13) is the increase of the convergence rate when the state is far away from the origin. Therefore, the integral of Eq. (13) gives the reaching time t_s.

This reaching time is:

$$t_s = \frac{p}{\alpha(p-q)} \ln \frac{\alpha x(0)^{\frac{p-q}{p}} + \beta}{\beta}. \tag{14}$$

Assumption 2. Equation (12) is used to design the first sliding surface, which is selected as:

$$s_1 = \dot{\theta}_e + \alpha\theta_e + \beta\theta_e^{q/p} = 0. \tag{15}$$

Equation (15) becomes:

$$\dot{\theta}_e = -\alpha\theta_e - \beta\theta_e^{q/p}. \tag{16}$$

According to Eq. (10) and Eq. (16), the following result is obtained:

$$\omega_r - \omega = -\beta\theta_e^{q/p} - \alpha\theta_e. \tag{17}$$

The control law is obtained:

$$\omega_c = \omega_r + \beta\theta_e^{q/p} + \alpha\theta_e. \tag{18}$$

The control law ω_c can take θ_e to zero in finite time t_e and the system Eq. (11) reaches the first sliding surface $s_1 = 0$. Then, the reaching time is:

$$t_e = \frac{p}{\alpha(p-q)} \ln \frac{\alpha\theta_e(0)^{\frac{p-q}{p}} + \beta}{\beta}. \tag{19}$$

Remark 1. The control law ω_c converge the disturbed angular velocity to the reference one. Then, the system Eq. (11) reaches $\omega_r \approx \omega + d\omega$ in the time t_e.

Proof 1. In order to ensure the stability of the system, the select Lyapunov function is given as:

$$V_{\theta_e} = \frac{1}{2}\theta_e^2. \tag{20}$$

The derivative is given by:

$$\dot{V}_{\theta_e} = \theta_e\dot{\theta}_e, \tag{21}$$

$$\dot{V}_{\theta_e} = -\beta\theta_e^{(q/p)+1} - \alpha\theta_e^2 \leq 0. \quad (22)$$

Remark 2. The derivative of V_{θ_e} is less than or equal to zero by taking into account that the parameters $\alpha \geq \beta$ and the parameters p and q satisfy the condition $\frac{q}{p} \prec \frac{\ln\alpha - \ln\beta}{\ln\theta_e} + 1$. Then, the system Eq. (11) converges asymptotically to the first sliding surface $s_1 = 0$.

3.2. Linear Velocity Controller Design

In the reaching time t_e, the system reaches the first sliding surface and the state θ_e tends to zero. Then, the system Eq. (11) becomes:

$$\dot{x}_e = \omega_r y_e + v_r - v - dv, \quad (23)$$

$$\dot{y}_e = -\omega_r x_e. \quad (24)$$

Assumption 3. From Eq. (23) and Eq. (24), the selected switching function is given as:

$$s_2 = x_e - y_e. \quad (25)$$

By designing the sliding mode control law, which leads the sliding surface s_2 to attain zero, it is interesting to consider the convergence of the state x_e to the state y_e and the two states converging to zero. In fact, the exponential reaching law is defined by the following equation:

$$\dot{s_2} = -G(t)Sign(s_2) - ks_2. \quad (26)$$

To eliminate the chattering, a continuous function replaces the sign function:

$$\dot{s}_2 = -G(t)\frac{\dot{s}_2}{\mid \dot{s}_2 \mid + \delta} - k\dot{s}_2, \quad (27)$$

where k, δ and $G(t)$ are positive parameters.

Equation (23), Eq. (24) and Eq. (25) are used and the result is obtained as:

$$\dot{s}_2 = \dot{x}_e - \dot{y}_e = \omega_r y_e + \omega_r x_e + v_r - v_c. \quad (28)$$

Using Eq. (27) and Eq. (28), the control law v_c is determined as:

$$v_c = v_r + \omega_r x_e + \omega_r y_e + G(t)\frac{s_2}{\mid s_2 \mid + \delta} + ks_2, \quad (29)$$

where

$$G(t) = \max \mid dv(t) \mid . \quad (30)$$

In Eq. (29), $G(t)$ is used in order to compensate the effect of disturbance $dv(t)$ and ensures the existing condition of the sliding mode and permits to avoid the chattering phenomenon.

Proof 2. For the stability analysis, the following Lyapunov function is considered:

$$V_l = \frac{1}{2}s_2^2. \quad (31)$$

The derivative of V_l is given by:

$$\dot{V}_l = s_2\dot{s}_2 = s_2(\omega_r x_e + \omega_r y_e + v_r - v_c - dv). \quad (32)$$

Using Eq. (29, the following equation is obtained:

$$\begin{aligned} \dot{V}_l &= s_2(-dv - G(t)\frac{s_2}{\mid s_2 \mid + \delta} - ks_2) = \\ &= -s_2 dv - G(t)\frac{s_2^2}{\mid s_2 \mid + \delta} - ks_2^2 \leq 0. \end{aligned} \quad (33)$$

To provide the stability, the fuzzy system is applied and the Lyapunov function V_l should be negative. Indeed, $dv(t)$ is time variant and in order to bring out the effect of uncertainties, $G(t)$ is time variant.

Remark 3. While the second surface $s_2 = 0$, the states x_e and y_e are equal and the states x and y are equal to the reference x_r and y_r respectively. Using this result and the square of the system Eq. (8), one deduces that $x_e = y_e = 0$. So, the asymptotic tracking stability is guaranteed.

3.3. Design of Fuzzy Control

The fuzzy system rules are used to estimate $G(t)$ and the control law of the linear velocity is designed [32] and [33]. A fuzzy system is used to achieve the parameter $\hat{G}(t)$ of the exponential reaching law v_c and the block diagram is shown in Fig. 2.
So, to design the fuzzy control, it is interesting to assure the condition of sliding mode, which is given by the following equation:

$$s_2\dot{s}_2 < 0. \quad (34)$$

If Eq. (34) is satisfied, then the system states will be on the second sliding surface.

Assumption 4. In this system, s_2 and $\dot{s_2}$ are the inputs and $G(t)$ is the output. The fuzzy sets of the inputs and output are the same and defined as follows: (NL, NM, Z, PM, PL).

Where, NL, NM, Z, PM, PL are linguistic words and presented as negative large, negative medium, zero, positive medium and positive large. To ensure the presence condition of sliding mode, the fuzzy rules are applied as:

- If S_2 is NL and if $\dot{S}_2$ is NL then $G(t)$ is NL.

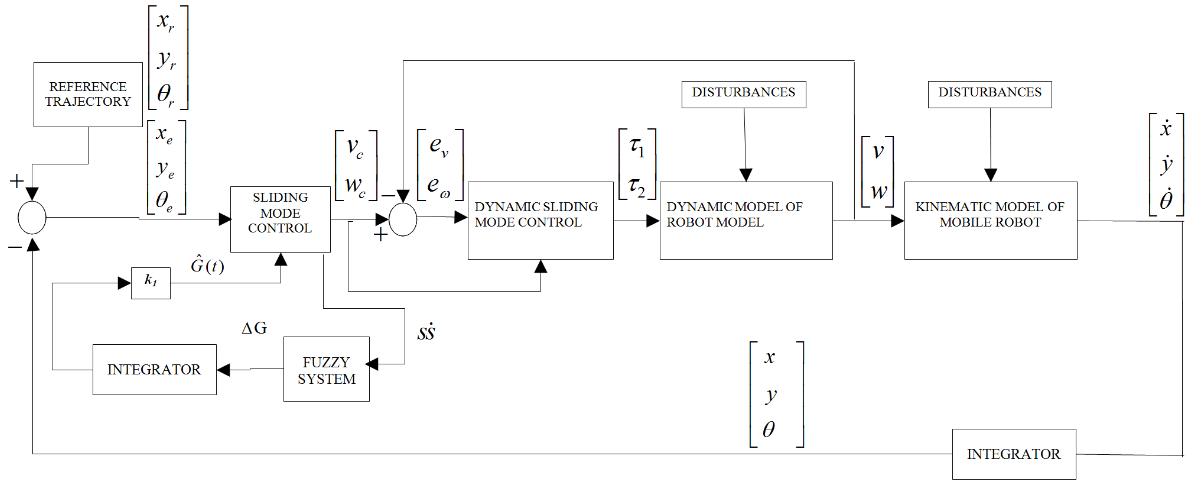

Fig. 2: Diagram of the control system.

- If S_2 is NM and if $\dot{S}_2$ is NM then $G(t)$ is NM.
- If S_2 is Z and if $\dot{S}_2$ is Z then $G(t)$ is Z.
- If S_2 is PM and if $\dot{S}_2$ is PM then $G(t)$ is PM.
- If S_2 is PL and if $\dot{S}_2$ is PL then $G(t)$ is PL.

The inputs membership function is shown in Fig. 3 and Fig. 4 respectively. Thus, the output membership function is given in Fig. 5.

The parameter $G(t)$ is estimated by:

$$\hat{G}(t) = k_1 \int_0^t G(t)dt, \tag{35}$$

where k_1 represents a gain.

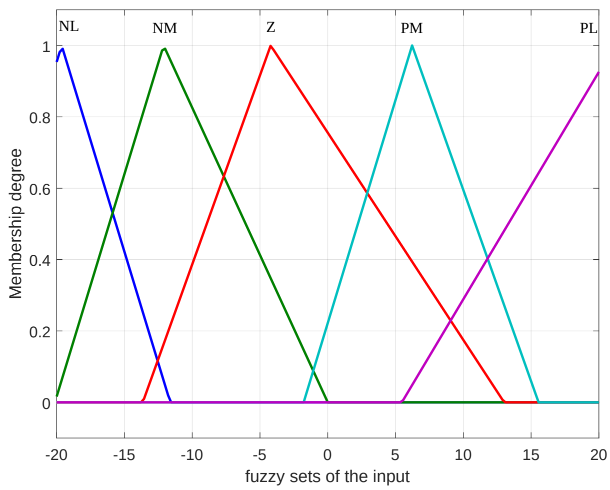

Fig. 3: Fuzzy sets of input function s_2.

So, the control law in Eq. (29) becomes a new fuzzy control law, which is indicated in this form:

$$v_c = v_r + \omega_r x_e + \omega_r y_e + k s_2 + \hat{G}(t)\frac{s_2}{\mid s_2 \mid + \delta}. \tag{36}$$

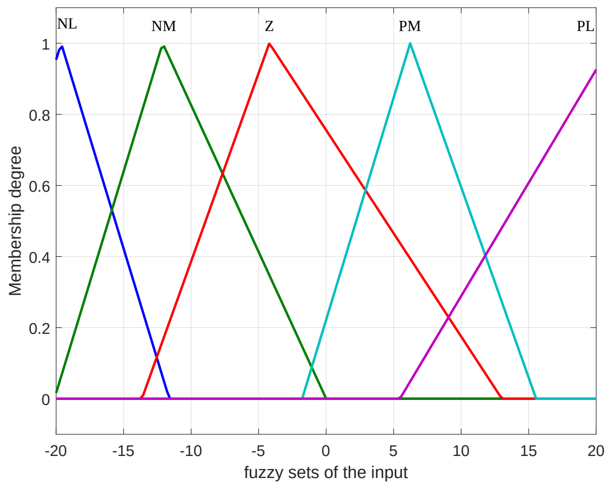

Fig. 4: Fuzzy sets of input function $\dot{s}_2$.

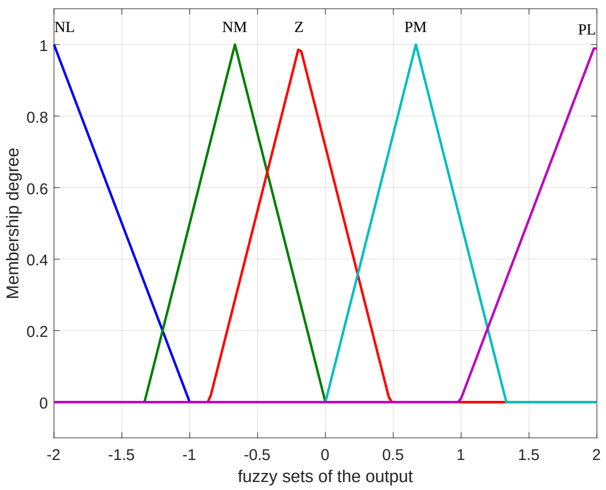

Fig. 5: Fuzzy sets of output function $G(t)$.

Proof 3. For the stability analysis, the derivative of Lyapunov function Eq. (31) is considered.

$$\dot{V} = -s_2 dv - \mid s_2 \mid \hat{G}(t) - ks_2^2, \tag{37}$$

where $\mid s_2 \mid \hat{G}(t) - ks_2^2 > -s_2 dv$.

Therefore, the control v_c of sliding surface s_2 converges the error states to zero with asymptotic stability. The general control law of kinematic model is given as follows:

$$V_c = \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} \omega_r + \beta\theta_e^{q/p} + \alpha\theta \\ v_r + \omega_r x_e + \omega_r y_e + ks_2 + \hat{G}(t)\frac{s_2}{|s_2|+\delta} \end{bmatrix}. \tag{38}$$

In fact if $v = v_c$ and $\omega = \omega_c$, then the system closed loop is asymptotically stable.

4. Dynamic Control

In this section, an exponential sliding mode control is used for the torque control τ in order to guarantee the convergence error V_e of the velocities.

$$\lim_{t\to\propto} V_e = \lim_{t\to\propto} \parallel V_c(t) - V(t) \parallel = 0. \tag{39}$$

The control τ of dynamic model is designed in order to take the actual velocities of the robot to the reference obtained with the kinematic controller.

4.1. Dynamic Model

Consider the dynamic model Eq. (5) and taking into account that the robot moves in horizontal plan, therefore gravitational vector, centripetal vector, Coriolis matrix, friction matrix and unknown disturbances become zero. The dynamic model Eq. (5) becomes as follows:

$$M(q)\dot{V} = \beta(q)\tau + R(t). \tag{40}$$

4.2. Dynamic Control Based on Exponential Sliding Mode

In this section, the exponential sliding mode control is used to force the dynamic model of the robot to guarantee the asymptotic stability. Thus, the velocity error is chosen as:

$$V_e = \begin{bmatrix} e_v & e_\omega \end{bmatrix}^T = \begin{bmatrix} v_c - v \\ \omega_c - \omega \end{bmatrix}. \tag{41}$$

The derivative velocity error is given as:

$$\dot{V}_e = \begin{bmatrix} \dot{v}_c - \dot{v} \\ \dot{\omega}_c - \dot{\omega} \end{bmatrix}. \tag{42}$$

The selected sliding surfaces are:

$$S = \begin{bmatrix} s_3 \\ s_4 \end{bmatrix} = \begin{bmatrix} v_c - v \\ \omega_c - \omega \end{bmatrix}. \tag{43}$$

The derivative sliding surfaces are:

$$\dot{S} = \begin{bmatrix} \dot{s}_3 \\ \dot{s}_4 \end{bmatrix} = \begin{bmatrix} \dot{v}_c - \dot{v} \\ \dot{\omega}_c - \dot{\omega} \end{bmatrix}. \tag{44}$$

Assumption 5. Using the exponential sliding mode control:

$$\dot{S} = \begin{bmatrix} -\varepsilon_1 \text{sgn}(s_3) - \mu_1 s_3 \\ -\varepsilon_2 \text{sgn}(s_4) - \mu_2 s_4 \end{bmatrix}, \tag{45}$$

where $\varepsilon = \begin{bmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix}$ and $\mu = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix}$ are positive constants.

Thus, combining Eq. (41) and Eq. (46), the dynamic control is given as:

$$\tau = \beta(q)^{-1} M(q)(\dot{V}_c + \varepsilon \text{sgn}(S) + \mu S). \tag{46}$$

Consider the dynamic model Eq. (40) with the disturbance and uncertainties $R(t)$, the result is given as [34]:

$$\dot{V} = M(q)^{-1}\beta(q)\tau + M(q)^{-1}R(t). \tag{47}$$

By choosing:

$$M(q)^{-1}\beta(q) = Q = (\hat{Q} + \Delta Q). \tag{48}$$

ΔQ represents the uncertainties and $\hat{Q}$ is the nominal term of the matrix. Therefore, $\varphi(t)$ is defined as the upper bound uncertainty, which is given as follows:

$$\Delta Q\tau + M(q)^{-1}R(t) = \varphi(t), \tag{49}$$

where $\mid \varphi(t) \mid \leq \phi$ and ϕ is constant positive parameter.

The control law is determined as:

$$\tau = \hat{Q}^{-1}(\dot{V}_c + \varepsilon \text{sgn}(S) + \mu S), \tag{50}$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \hat{Q}^{-1}\left(\begin{bmatrix} \dot{v}_c \\ \dot{\omega}_c \end{bmatrix} + \begin{bmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix} \begin{bmatrix} \text{sgn}(s_3) \\ \text{sgn}(s_4) \end{bmatrix} + \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \begin{bmatrix} s_3 \\ s_4 \end{bmatrix} \right). \tag{51}$$

In order to avoid the chattering phenomenon created by the sign function, a quasi-sliding mode function is applied:

$$\dot{S} = \begin{bmatrix} -\varepsilon_1 \frac{s_3}{\text{abs}(s_3)+\delta_1} - \mu_1 s_3 \\ -\varepsilon_2 \frac{s_4}{\text{abs}(s_4)+\delta_2} - \mu_1 s_4 \end{bmatrix}. \tag{52}$$

The control law of the dynamic system is:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \hat{Q}^{-1}\left(\begin{bmatrix} \dot{v}_c \\ \dot{\omega}_c \end{bmatrix} + \begin{bmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix} + \begin{bmatrix} \frac{s_3}{\text{abs}(s_3)+\delta_1} \\ \frac{s_4}{\text{abs}(s_4)+\delta_2} \end{bmatrix} \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \begin{bmatrix} s_3 \\ s_4 \end{bmatrix} \right). \tag{53}$$

Proof 4. The selection of Lyapunov function is as follows [35]:

$$V_d = \frac{1}{2} S^T S. \quad (54)$$

The derivative of the Lyapunov function V_d is obtained.

$$\dot{V}_d = S^T \dot{S} = s_3 \dot{s}_3 + s_4 \dot{s}_4. \quad (55)$$

Equation (55) becomes as follows:

$$\begin{aligned} \dot{V}_d = S^T \dot{S} = -\varepsilon_1 \frac{s_3^2}{\mathrm{abs}(s_3) + \delta_1} - \mu_1 s_3^2 \\ -\varepsilon_2 \frac{s_4^2}{\mathrm{abs}(s_4) + \delta_2} - \mu_2 s_4^2 \leq 0, \end{aligned} \quad (56)$$

where diag $(\varepsilon_1,\ \varepsilon_2) > 0$, diag $(\mu_1,\ \mu_2) > 0$ and δ_1, $\delta_2 > 0$.

Remark 4. The parameter diag $(\mu_1,\ \mu_2)$ is taken in order to compensate the effect of disturbances and uncertainties of the system.

5. Simulation Results

In this section, to show the efficacy of the control law, simulation works under Matlab environment are realized, and three different trajectories (circular, sinusoidal and specific) are considered. The mobile robot parameters used for simulation are given as follows: m=4 kg, I=3 kg·m^2, R_a=0.03 m and L=0.15 m.

The desired parameters of control are selected as below v_r=2 m·s^{-1} and ω_r=2 rad·s^{-1}. The desired trajectory of the robot is:

$$\begin{cases} x_r & r\cos(\omega_r t) = \cos(2t) \\ y_r & r\sin(\omega_r t) = \sin(2t) \\ & \theta_r = \omega_r t = 2t. \end{cases} \quad (57)$$

The controller parameters are chosen arbitrarily. $p = 10$, $q = 9$, $k = 30$, $K_1 = 250$, $\alpha = 50$, $\beta = 50$, $\delta = 0.08$.

A limited periodic disturbance term is inserted between time $4 < t < 5$ and considered as follows:

$$\begin{cases} dv = \sin(t - \pi) \\ d\omega = 2\sin(t - \pi). \end{cases} \quad (58)$$

The parameters of the dynamic controller are selected as: $\varepsilon_1 = 80, \varepsilon_2 = 80, \mu_1 = 30, \mu_2 = 30, \delta_1 = 0.95$and $\delta_2 = 0.95$ and the dynamic disturbances, introduced between four and five seconds (4 s< t <5 s) are considered:

$$E(t) = [2.5\sin(t - \pi) \quad 1.5\sin(t - \pi)]. \quad (59)$$

The initial position and orientation errors are given as (2 m, 1 m) and ($\pi/6$ rad) respectively. Figure 6 represents the control signals v_c and ω_c of the kinematic model.

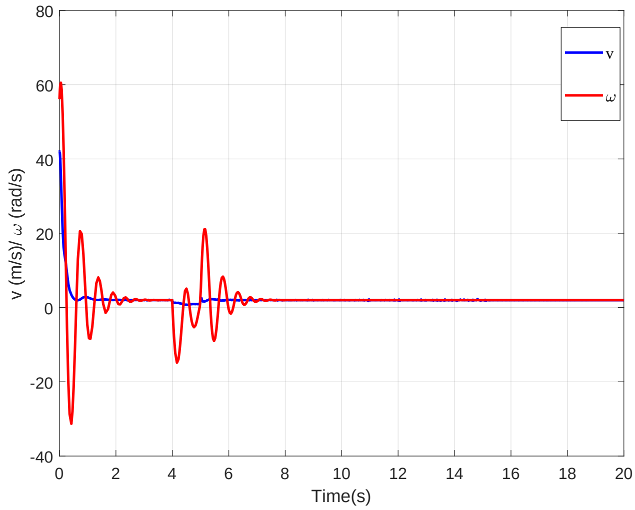

Fig. 6: Control law of the signals v_c and ω_c.

It can be seen that the actual linear and angular velocities of the proposed control can reach the desired velocities in short time.

By using the kinematic and dynamic controllers of Eq. (38) and Eq. (53), the simulation results of a circular trajectory tracking are shown in Fig. 7.

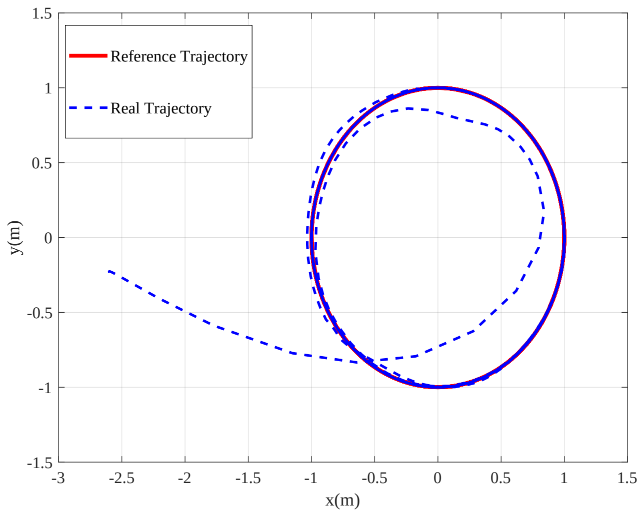

Fig. 7: Circular trajectory tracking of the mobile robot.

Hence, the mobile robot can achieve the circular trajectory rapidly in short time among $t = 2$ s in presence of disturbances and the asymptotic stability of the robot is assured. This is the main advantage of the proposed control law in term of error states convergence to zero and the elimination of the disturbance effect. On the other side in [36], the robot cannot converge completely to the reference and the error states cannot stabilize fully to zero due of some perturbation.

Figure 8 represents the trajectories tracking errors of the states x_e and y_e of the system ρ_e.

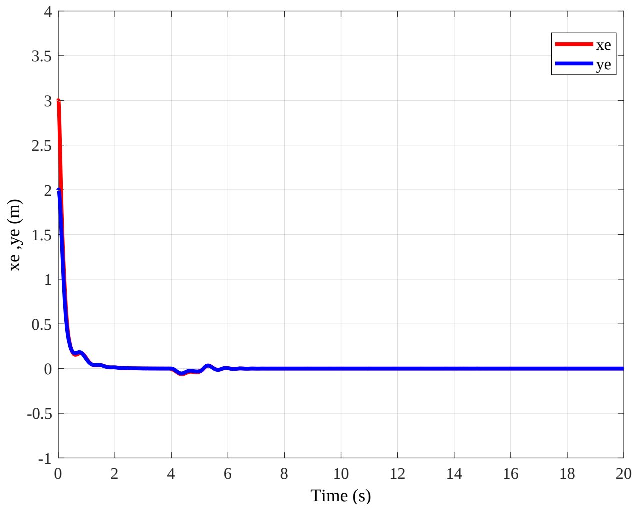

Fig. 8: Position tracking errors of the states x_e and y_e.

The orientation error θ_e is illustrated in Fig. 9.

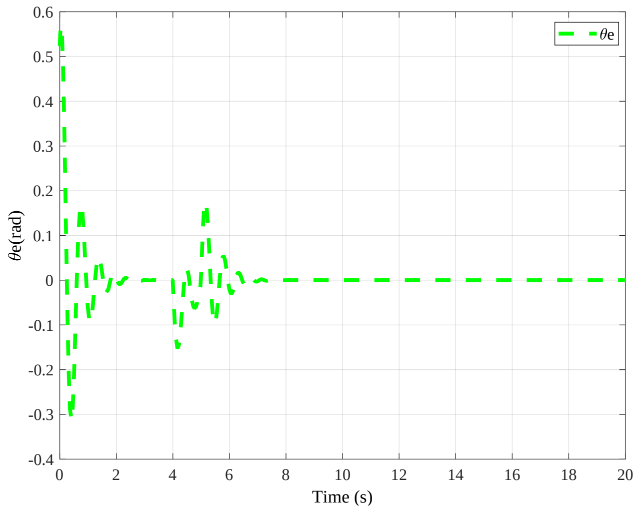

Fig. 9: Orientation tracking error θ_e.

By using the dynamic controller, the torques and the velocity errors are shown in Fig. 10 and Fig. 11 respectively.

For the sinusoidal trajectory, the same initial position and orientation errors are considered, but the reference posture is selected as follows:

$$\begin{cases} x_t = t \\ y_r = \sin(2t) \\ \theta_r = \omega_r t = 2t \end{cases} . \tag{60}$$

The control signals v_c and ω_c are illustrated in Fig. 12.

The simulation results for a sinusoidal trajectory tracking are shown in Fig. 13. It can be seen that

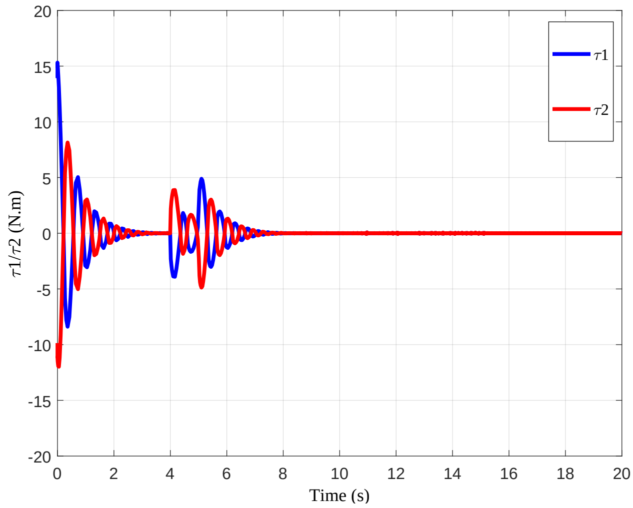

Fig. 10: Generated torques τ_1 and τ_2.

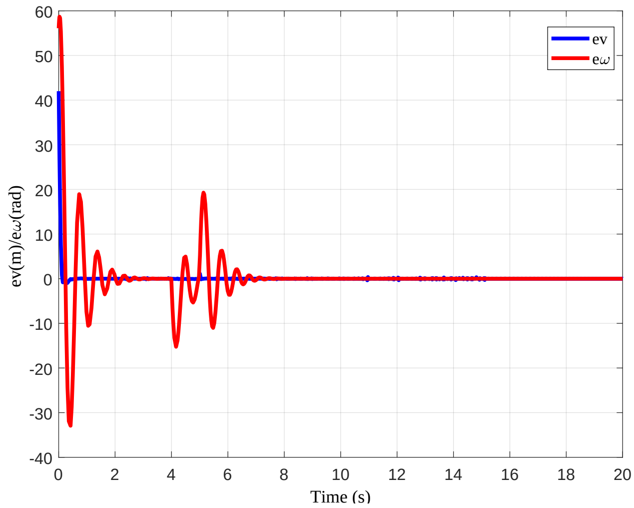

Fig. 11: Velocity errors e_v and e_ω.

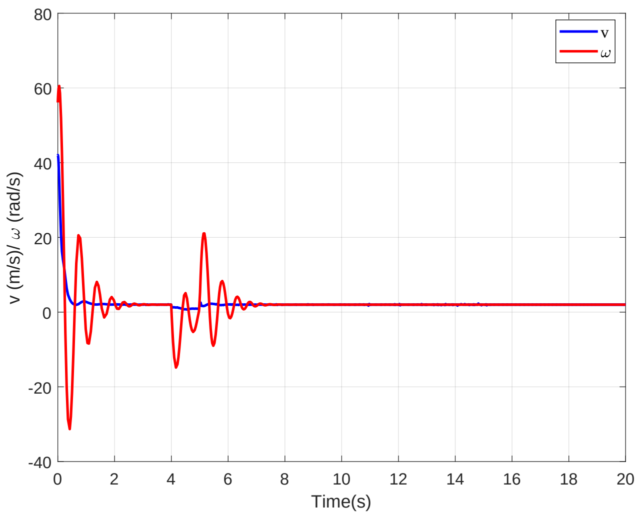

Fig. 12: Control law of the signals v_c and ω_c.

the mobile robot converges to the reference trajectory. Figure 14 represents the trajectory tracking errors of the states x_e and y_e of the system ρ_e. The orientation θ_e is illustrated in Fig. 15.

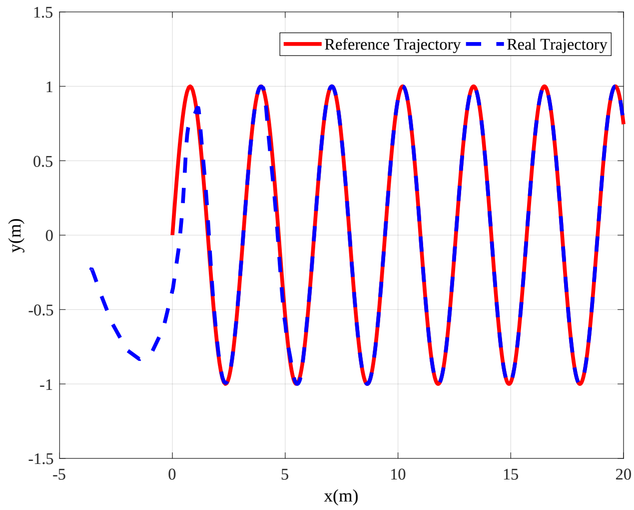

Fig. 13: Sinusoidal trajectory tracking.

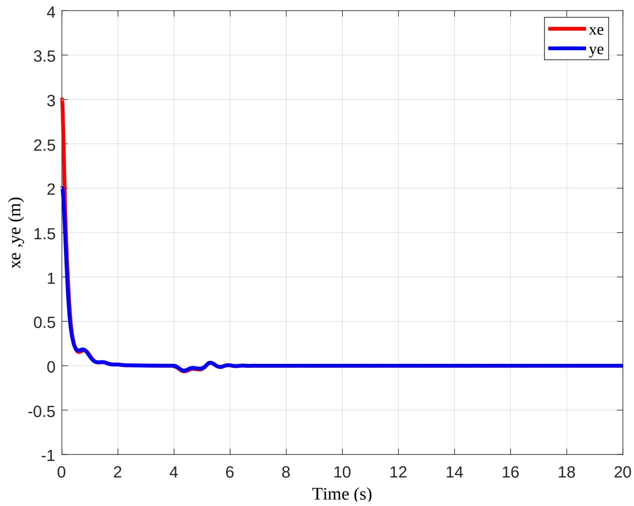

Fig. 14: Position tracking errors of the states x_e and y_e.

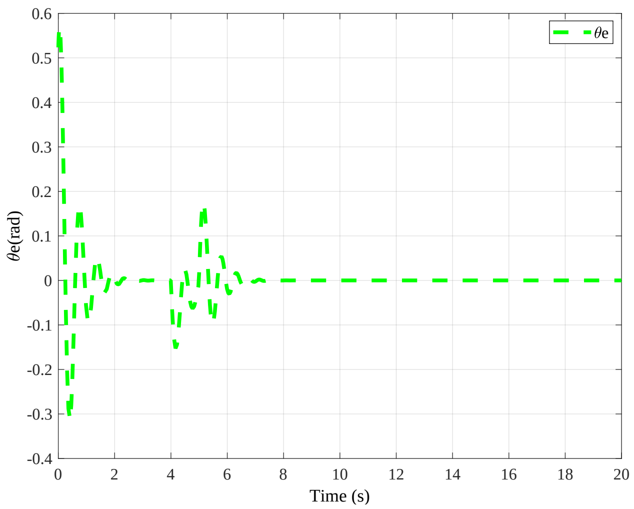

Fig. 15: Orientation tracking error θ_e.

The torques and the error velocities are shown in Fig. 16 and Fig. 17 respectively.

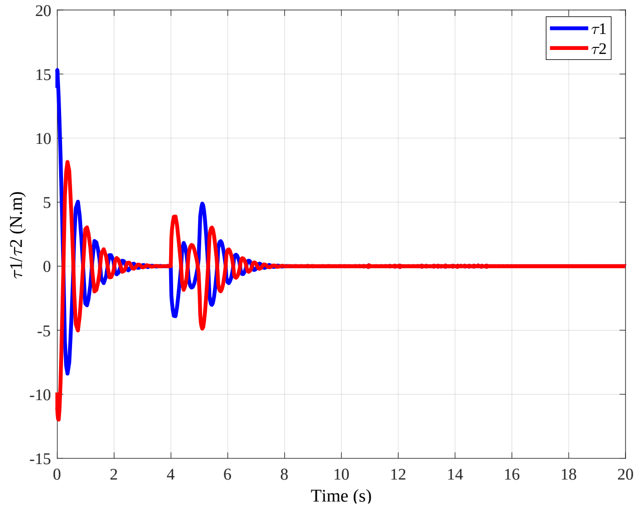

Fig. 16: Generated torques τ_1 and τ_2.

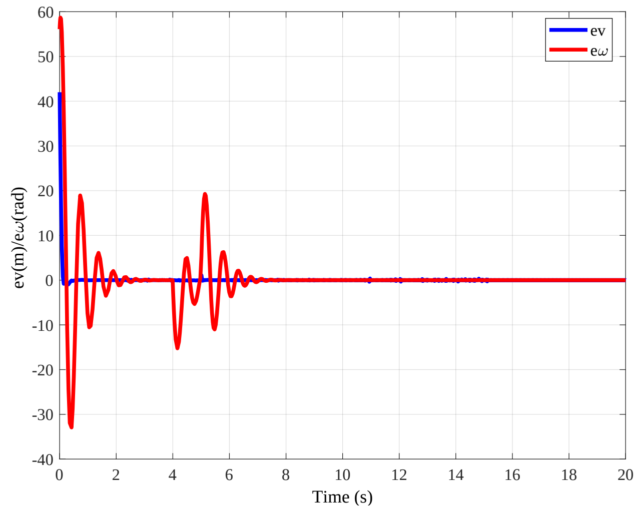

Fig. 17: Velocity errors e_v and e_ω.

The torques obtained with the proposed controller in presence of disturbances converges to zero. Thus, the error velocities converge asymptotically to zero before and after inserting the external disturbances.

For the specific trajectory, the followed values (3 m, 2 m $\pi/6$ rad), for the initial position and orientation error are considered.

$$\begin{cases} x_r = \cos t \\ y_r = \sin 2t \\ \theta_r = \omega_r t = 2t \end{cases} . \tag{61}$$

The control signals v_c and ω_c are illustrated in Fig. 18.

The simulation results of the specific trajectory are shown in Fig. 19.

Figure 20 represents the error states of x_e and y_e. The orientation error θ_e is showed in Fig. 21.

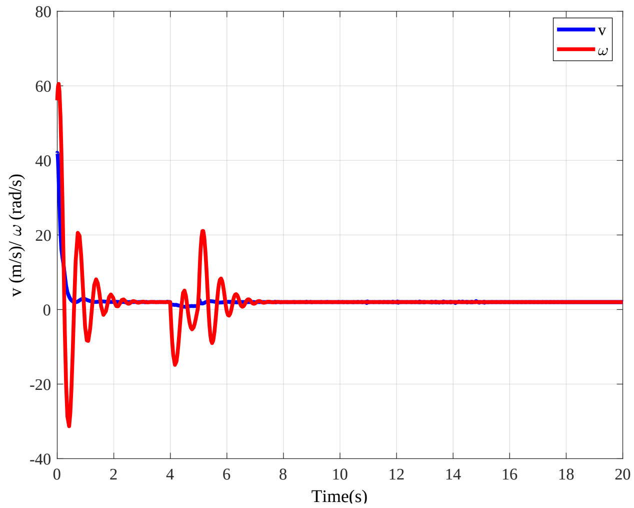

Fig. 18: Control law of the signals v_c and ω_c.

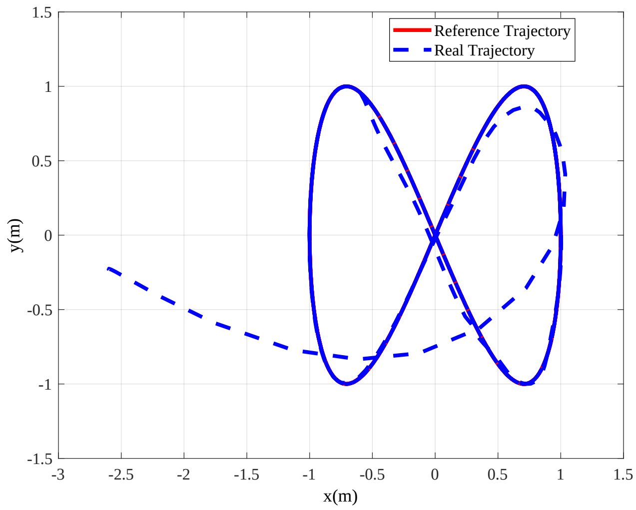

Fig. 19: Specific trajectory tracking.

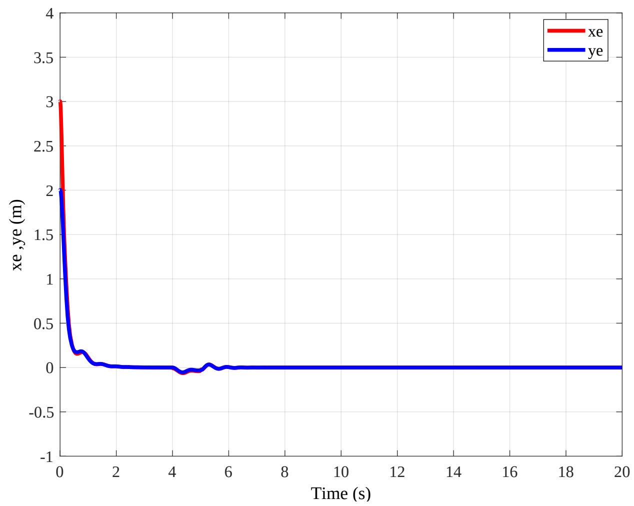

Fig. 20: Position tracking errors.

The torques and error velocities are shown in Fig. 22 and Fig. 23.

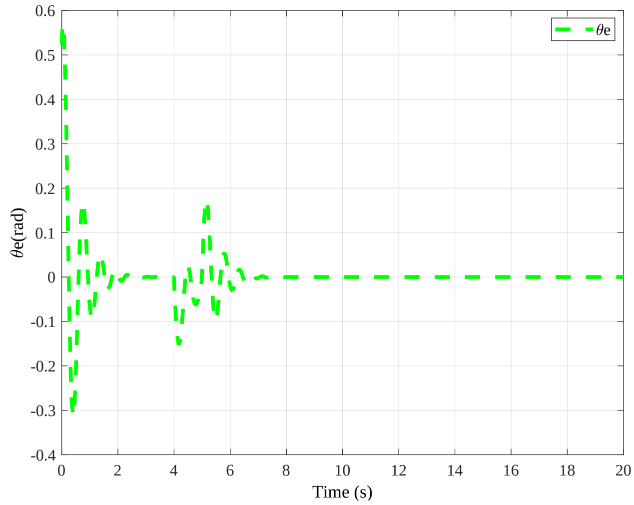

Fig. 21: Orientation tracking error θ_e.

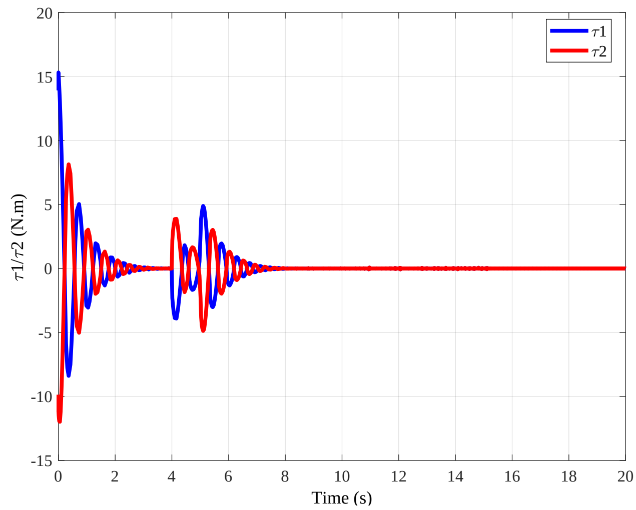

Fig. 22: Generated torques τ_1 and τ_2.

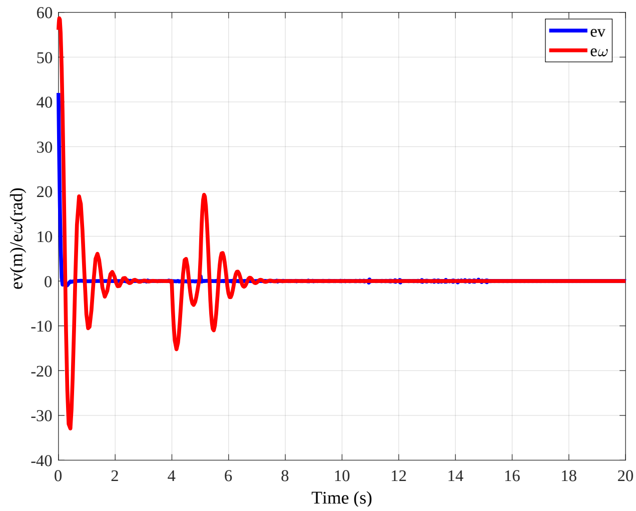

Fig. 23: Velocity errors e_v and e_ω.

The comparison of each selected trajectory is given in the Tab. 1, using the convergent time and the mean error which is the average error using Eq. (62):

$$ME = \sum_{k=1}^{N} \frac{E(k)}{N}. \tag{62}$$

Tab. 1: Convergent time and mean error.

	Trajectories		
	Circular	Sinusoidal	Specific
Convergenttime before disturbances(s)	2	3	2
Convergent time after disturbances(s)	7	7	7
Mean position error of x_e(m)	0.1216	0.1218	0.1893
Mean position error of y_e (m)	0.0682	0.0685	0.1315
Mean orientation error of θ_e (m)	0.0293	0.0292	0.0282

Figure 7, Fig. 13 and Fig. 19 show the simulation results for different initial posture errors under fuzzy sliding mode control. It can easily be seen that the tracking errors x_e, y_e and θ_e tend to zero using the control laws Eq. (38) and Eq. (53).

Comparing the results achieved in this work, and those obtained with other works, it can be seen that the convergence of error states in this paper is rapid with $t = 2$ s, contrariwise to [37] where $t = 6$ s.

It can be viewed that the system is affected by the perturbation, but the control law v_c and the control torque τ can return the mobile robot to its reference. In Fig. 11, Fig. 17 and Fig. 23, the error between the linear velocities v and v_c and the angular velocities ω and ω_c converge to zero in short time after the disturbances have been inserted in the system.

The proposed controller can correct the deviation caused by some kind of perturbations, and the system stays asymptotically stable. Furthermore, the chattering phenomenon is avoided.

The control obtained based on fuzzy sliding mode control has the ability to remove some kind of possible bounded disturbances by selecting the appropriate value of the parameter $G(t)$, which is a function of the fuzzy sets of the second sliding surface and its derivative, therefore the selected parameter $G(t)$ can delete the inserted disturbances.

All the simulation results indicate that the posture errors converge to zero in presence of external disturbances in a very short time. In other words, the results verify that the control method is robust to break out the external disturbances and uncertainties in case of many trajectories (sinusoidal, circular and specific).

6. Conclusion

In this paper, a Global Fast Terminal Sliding Mode (GFTSM) and fuzzy logic approach with exponential sliding mode control is presented. A GFTSM is designed for the angular velocity and implemented in order to take the angle error to zero in a finite time, which is indicated in Fig. 9, Fig. 15 and Fig. 21. The convergence time is almost 2 seconds before inserting the disturbances, and $t = 3$ s after introducing the disturbances.

The presented control law is based on exponential reaching law with fuzzy system and an integrator gain for the linear velocity in order to take the error states xe and ye to zero, thus the simulation results show that the two states converge in the time t=1.5 s. The dynamic control based on exponential sliding mode control aims to converge the velocity errors to zero between the real velocities and the velocities obtained with kinematic controller; therefore the convergent time of the velocity error is almost 2 seconds.

These control laws of linear and angular velocity can assure the asymptotical stability of the system by applying the Lyapunov theory, and proves that the controller is stable for any combination of the error states and can bring out the chattering phenomenon. The advantage of this control law is to eliminate the uncertainties and the external disturbances due to kinematic model and dynamic model, such that the error states of the robot converge to zero.

The undertaken simulation works show clearly the efficacy of this approach. This technique has been proved for nonholonomic mobile robots in trajectories tracking with different configurations as circular, sinusoidal and specific trajectories.

References

[1] KHATIB, M., H. JAOUNI, R. CHATILA and J. P. LAUMOND. Dynamic path modification for car-like nonholonomic mobile robots. In: *Proceedings of International Conference on Robotics and Automation*. Albuquerque: IEEE, 1997, pp. 2920–2925. ISBN 0-7803-3612-7. DOI: 10.1109/ROBOT.1997.606730.

[2] RAMIREZ, G. and S. ZEGHLOUL. A New Local Path Planner for Nonholonomic Mobile Robot Navigation in Cluttered Environments. In: *IEEE International Conference on Robotics and Automation*. San Francisco: IEEE, 2000, pp. 2058–2063. ISBN 0-7803-5886-4.

[3] RAZVAN, S. and U. NUNES. Trajectory Planning and Sliding Mode Control Based Trajec-

tory Tracking for Cybercars. *Integrated Computer-Aided Engineering*. 2007, vol. 14, no. 1, pp. 33–47. ISSN 1875-8835.

[4] KHENNOUF, H. and C. C. DE WIT. Quasi-Continuous Exponential Stabilizers for Nonholonomic Systems. *IFAC Proceedings Volumes*. 1996, vol. 29, iss. 1, pp. 2448–2453. ISSN 1474-6670. DOI: 10.1016/s1474-6670(17)58041-6.

[5] DINUZZO F. and A. FERRARA. Higher order sliding mode controllers with optimal reaching. *IEEE Transactions on Automatic Control*. 2009, vol. 54, iss. 9, pp. 2126–2136. ISSN 0018-9286. DOI: 10.1109/tac.2009.2026940.

[6] KEIGHOBADI, J. and Y. MOHAMADI. Fuzzy Sliding Mode Control of non-holonomic Wheeled Mobile Robot. In: *IEEE 9th International Symposium on Applied Machine Intelligence and Informatics*. Smolenice: IEEE, 2011, pp. 273–278. ISBN 978-1-4244-7429-5. DOI: 10.1109/SAMI.2011.5738888.

[7] YANG J. M. and J. H. KIM. Sliding mode control for trajectory tracking of nonholonomic wheeled mobile robots. *Transactions on Robotics and Automation*. 1999, vol. 15, iss. 3, pp. 578–587. ISSN 1042-296X. DOI: 10.1109/70.768190.

[8] DUNG, N. M., V. H. DUY, N. T. PHUONG, S. B. KIM and M. S. OH. Two Wheeled Welding Mobile Robot for Tracking a Smooth Curved Welding Path Using Adaptive Sliding-Mode Control Technique. *International Journal of Control Automation and Systems*. 2007, vol. 5, iss. 3, pp. 283–294. ISSN 1598-6446.

[9] UTKIN, V. I. Variable structure systems with sliding modes. *IEEE Transactions on Automatic Control*. 1977, vol. 22, iss. 2, pp. 212–222. ISSN 0018-9286. DOI: 10.1109/TAC.1977.1101446.

[10] ORLOV, Y. *Discontinuous systems: Lyapunov analysis and robust synthesis under uncertainty conditions.*. London: Springer-Verlag, 2010. ISBN 978-1-84800-984-4.

[11] DONG, C. S. T., H. H. VO, T. C. TRAN, P. BRANDSTETTER and P. SIMONIK. Application of Sensorless Sliding Mode Observer in Control of Induction Motor Drive. *Advances in Electrical and Electronic Engineering*. 2018, vol. 15, iss. 5, pp. 747–753. ISSN 1804-3119. DOI: 10.15598/aeee.v15i5.2626.

[12] PARK, M., D. CHWA and S. HONG. Decoupling Control of A Class of Underactuated Mechanical Systems Based on Sliding Mode Control. In: *SICE-ICASE International Joint Conference*. Busan: IEEE, 2006, pp. 806–810. ISBN 89-950038-4-7. DOI: 10.1109/SICE.2006.315338.

[13] IBRAHIM, A. E. B. Wheeled Mobile Robot Trajectory Tracking using Sliding Mode Control. *Journal of Computer Science*. 2016, vol. 12, iss. 1, pp. 48–55. ISSN 1549-3636. DOI: 10.3844/jcssp.2016.48.55.

[14] ZHANG, J., B. ZHOU, S. WEI and Y. SONG. Study on sliding mode trajectory tracking control of mobile robot based on the Kalman filter. In: *IEEE International Conference on Information and Automation*. Ningbo: IEEE, 2016, pp. 1195–1199. ISBN 978-1-5090-4102-2. DOI: 10.1109/ICInfA.2016.7832001.

[15] HONG, M. Trajectory Tracking Control of Robot via a Fuzzy Sliding-mode Controller. *Advanced Materials Research*. 2012, vol. 433, iss. 1, pp. 4154–4158. ISSN 1662-8985. DOI: 10.4028/www.scientific.net/AMR.433-440.

[16] CASTILLO, O., L. T. AGUILAR and S. CARDENAS. Fuzzy logic tracking control for unicycle mobile robots. *Engineering Letters*. 2006, vol. 13, iss. 1, pp. 73–77. ISSN 1816-0948.

[17] CHEN, C. Y., T. S. LI and Y. YEH. EP-based kinematic control and adaptive fuzzy sliding-mode dynamic control for wheeled mobile robots. *Information Sciences*. 2009, vol. 179, iss. 2, pp. 180–195. ISSN 0020-0255. DOI: 10.1016/j.ins.2008.09.012.

[18] KAZRAJI, S. M. and M. B. B. SHARIFIAN. Direct Thrust Force and Flux Control of a PM-Linear Synchronous Motor Using Fuzzy Sliding-Mode Observer. *Advances in Electrical and Electronic Engineering*. 2015, vol. 13, iss. 1, pp. 1–9. ISSN 1804-3119. DOI: 10.15598/aeee.v13i1.999.

[19] LIU, X., H. ZHAO and H. LIU. Adaptive Fuzzy Non-singular Terminal Sliding Mode Control for a Class of Uncertain Nonlinear Systems. *Journal of Information & Computational Science*. 2013, vol. 10, iss. 1, pp. 1229–1236. ISSN 1746-7659.

[20] NEKOUKAR, V. and A. ERFANIAN. Adaptive fuzzy terminal sliding mode control for a class of MIMO uncertain nonlinear systems. *Fuzzy Sets and Systems*. 2011, vol. 179, iss. 1, pp. 34–49. ISSN 0165-0114. DOI: 10.1016/j.fss.2011.05.009.

[21] PARK, K. and T. TSUJI. Terminal sliding mode control of second-order nonlinear uncertain systems. *International Journal of Robust and Nonlinear Control*. 1999, vol. 9, iss. 11, pp. 769–780. ISSN 1099-1239. DOI: 10.1002/(SICI)1099-1239(199909)9:11<769::AID-RNC435>3.0.CO;2-M.

[22] MARTINS, N. A., M. ALENCAR, W. C. LOMBARDI, D. W. BERTOL, E. R. PIERI and H. F. FILHO. Trajectory tracking of a wheeled mobile robot with uncertainties and disturbances: proposed adaptive neural control. *Control and Cybernetics*. 2015, vol. 44, no. 1, pp. 47–98. ISSN 0324-8569.

[23] UTKIN, V., J. GULDNER and J. SHI. *Sliding Mode Control in Electro-Mechanical Systems*. 2nd ed. Boca Raton: CRC Press, 2009. ISBN 978-1-420-06560-2.

[24] MARTINS, N. A., E. S. ELYOUSSEF, D. W. BERTOL, E. R. DE PIERI, U. F. MORENO and E. B. CASTELAN. Nonholonomic Mobile Robot with Kinematic Disturbances in the Trajectory Tracking: A Variable Structure Controller. *Learning and Nonlinear Models*. 2010, vol. 8, iss. 1, pp. 23–40. ISSN 1676-2789. DOI: 10.21528/LNLM-vol8-no1-art2.

[25] DIXON, W. E., D. M. DAWSON and E. ZERGEROGLU. Tracking and Regulation Control of a Mobile Robot System With Kinematic Disturbances: A Variable Structure-Like Approach. *Journal of Dynamic Systems, Measurement, and Control*. 2000, vol. 122, iss. 4, pp. 616–623. ISSN 0022-0434. DOI: 10.1115/1.1316795.

[26] CORRADINI, M. L., T. LEO and G. ORLANDO. Robust stabilization of a mobile robot violating the nonholonomic constraint via quasi-sliding modes. In: *Proceedings of the 1999 American Control Conference*. San Diego: IEEE, 1999, pp. 3935–3939. ISBN 0-7803-4990-3. DOI: 10.1109/ACC.1999.786255.

[27] MARTINS, N. and R. DE PIERI. Trajectory Tracking of a Nonholonomic Mobile Robot with Kinematic Uncertainties and Disturbances: An Adaptive Variable Structure Controller. In: *16th Mediterranean Conference on Control and Automation*. Ajaccio: IEEE, 2008, pp. 315–320. ISBN 978-1-4244-2505-1.

[28] LEE, T. C., K. T. SONG, C. H. LEE and C. C. TENG. Tracking control of unicycle-modeled mobile robots using a saturation feedback controller. *Transactions on Control Systems Technology*. 2001, vol. 9, iss. 2, pp. 305–318. ISSN 1063-6536. DOI: 10.1109/87.911382.

[29] LEE, J. H., C. LIN, H. LIM and J. M. LEE. Sliding mode control for trajectory tracking of mobile robot in the RFID sensor space. *International Journal of Control, Automation and Systems*. 2009, vol. 7, iss. 3, pp. 429–435. ISSN 1598-6446. DOI: 10.1007/s12555-009-0312-7.

[30] KOUBAA, Y., M. BOUKATTAYA and T. DAMMAK. Adaptive Sliding-Mode Dynamic Control For Path Tracking of Nonholonomic Wheeled Mobile Robot. *Journal of Automation & Systems Engineering*. 2015, vol. 9, iss. 2, pp. 119–131. ISSN 1112-8542.

[31] SHAO, J. X., Y. D. ZHAO, D. E. KIM and J. M. LEE. Trajectory Tracking by Terminal Sliding Mode Control for a Three-Wheeled Mobile Robot. In: *International Conference on Intelligent Robotics and Applications*. Cham: Springer, 2017, pp. 215–225. ISBN 978-3-319-65297-9. DOI: 10.1007/978-3-319-65298-6_20.

[32] WANG, L. X. Stable adaptive fuzzy control of nonlinear systems. *Transactions on Fuzzy Systems*. 1993, vol. 1, iss. 2, pp. 146–155. ISSN 1063-6706. DOI: 10.1109/91.227383.

[33] LIU, J. and X. WANG. *Advanced sliding mode control for mechanical systems: design, analysis and MATLAB simulation*. Beijing: Tsinghua University Press, 2012. ISBN 978-3-642-20907-9.

[34] MOHAGHEGHI, A., F. SHABANINIA and M. SALIMIFARD. Fuzzy Logic & Fuzzy Sliding Mode Tracking Control of Non-holonomic Unicycle Wheeled Mobile Robots. In: *21st Iranian Conference on Electrical Engineering*. Mashhad: IEEE, 2013, pp. 1–6. ISBN 978-1-4673-5634-3. DOI: 10.1109/IranianCEE.2013.6599538.

[35] YU-YAO, H. and Y. MAO-DE. *Nonlinear control theory and application*. Xi-an: Xian Electronic Science University of Technology Press, 2007.

[36] ESMAEILI, N., A. ALFI and H. KHOSRAVI. Balancing and Trajectory Tracking of Two-Wheeled Mobile Robot Using Backstepping Sliding Mode Control: Design and Experiments. *Journal of Intelligent & Robotic Systems*. 2017, vol. 87, iss. 3, pp. 601–613. ISSN 0921-0296. DOI: 10.1007/s10846-017-0486-9.

[37] PENG, S. and W. SHI. Adaptive Fuzzy Integral Terminal Sliding Mode Control of a Nonholonomic Wheeled Mobile Robot. *Mathematical Problems in Engineering*. 2017, vol. 2017, iss 1, pp. 1–12. ISSN 1024-123X. DOI: 10.1155/2017/3671846.

About Authors

Walid BENAZIZA is currently a Ph.D. student at the Electronics Department of Batna 2 University, Algeria. He received the engineer degree and Magister degree in Control from Batna University, Algeria respectively in June 2012 and Mars 2015. His research interests include Robotics and Artificial intelligence, linear and nonlinear systems control, dynamics modelling, soft computing and simulation.

Noureddine SLIMANE received the Engineer degree in Electronics in 1979 from National Polytechnic School of Algiers, M.Sc. degree and Ph.D. in Industrial Control and Robotics from Batna University, Algeria, in 1995 and 2005 respectively. Currently he is an Associate Professor in Automation and Robotics and responsible of Robotics and Artificial Intelligence Doctorate in Batna 2 University. His research interests include autonomous systems, intelligent and robust control, adaptive and optimal control, automation and robotics.

Ali MALLEM is currently a Ph.D. student at the Electronics Department of Batna 2 University, Algeria and Assistant in Khenchela University since September 2015. He received the engineer degree in Control in 2008 and the Magister degree in Robotics in 2011 from Batna University, Algeria. His research interests include Robotics and Artificial intelligence, robust control.