MODELLING THE LOAD TORQUES OF ELECTRIC DRIVE FOR POLYMERIZATION PROCESS

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Summary The problems of mathematical modelling the load torques on shaft of driving motor designed for applications in polymerization reactors are presented in the paper. The real load of polymerization drive is determined as a function of angular velocity. Mentioned function results from friction in roll-formed slide bearing as well as from friction of ethylene molecules with mixer arms in polymerization reactor chamber. Application of mathematical formulas concerning the centrifugal ventilator is proposed to describe the mixer in reactor chamber. The analytical formulas describing the real loads of polymerization drive are applied in mathematical modelling the power unit of polymerization reactor with specially designed induction motor. The numerical analysis of transient states was made on the basis of formulated mathematical model. Examples of transient responses and trajectories resulting from analysis are presented in the paper.

1. INTRODUCTION

The polymerization reactors play a significant role in production line of polyethylene. The driving motor connected with mixer works inside the reactor chamber. Operating conditions of motor are extraordinary due to the necessity of keeping a constant temperature in reactor chamber under ethylene atmosphere and working pressure of 2800×10^5 Pa [1 – 6]. The driving motor adapted for vertical work and located in the upper part of reactor chamber has specific dimensions because of fixing in a socket. The power grid or motor-generator set or frequency converter supplies the motor [7, 8]. Supply systems are often damaged due to exceptional working conditions, e.g. the feed of motor via specially designed electrodes.

A non-typical power unit designed for process of polymerization is characterized by the following extremely difficult operating conditions [9]:
- location of the 55 kW induction motor in closed tubular seat with diameter of 302 mm and total length of 919 mm;
- impossibility of application of both ventilation in the motor and conventional power supply due to location of motor directly in reactor chamber under pressure of 2800×10^5 Pa;
- vertical single-point suspension of motor together with the mixer; upper bearing plays a role of rotor alignment in stator; the choice of vertical system for motor operation was caused by construction of suspension system as well as assembly possibilities;
- the operation of vertical single-point suspension with application of self-aligning non-lubricate slide bearing containing the large-size rings made of sintered carbides; the ethylene streams cool the slide bearing via the ventilating ducts within external casing of bearing;
- the supply of stator winding via the set of spring electrical socket together with electrode ensuring a blockade of operating pressure in reactor chamber;
- a specially constructed electrical socket supplying the stator winding; this construction is fixed on tubular frame of motor and allows working during torsional vibration of the system in plane perpendicular to motor axis;
- reliability and stability of power supply based on frequency converter; possibility of continuous monitoring the working motor taking into consideration the parameters of supply voltage mainly the variable frequency of that; the continual measurements are necessary to ensure the processing parameters.

The ethylene atmosphere in the neighbourhood of slide bearing causes random occurrence of polymerization sockets during the drive operation. The random occurrence of polymerization sockets when the slide bearing is surrounded by the ethylene atmosphere is a frequent phenomenon during work of polymerization reactors. The polymerization sockets may cause significant change of operating conditions in reactor chamber together with silting of motor air-gap. It can cause overload of driving motor. Moreover, silting of both the motor air-gap and other rotating parts of drive is disadvantageous because of local increase of temperature what additionally speeds up an expansion of polymerization sockets [2].

The extreme working conditions of above described motors working in polymerization reactors forced the necessity of developing the new prototypes of more resistant specially designed induction motors. The motors SAR-55/1500/09 are shown in Fig. 1, whereas a lower bearing of prototype SAR-55/1500/09/P2 is given in Fig. 2 [3].

A load torque caused by friction in slide bearing is connected with application of vertical system for motor operation. A technical test stand designed in order to measure a load torque caused by friction in bearing was made and experimental results were obtained for specified working curvature of bearing [2]. These results allow formulating the analytical dependences describing load variability.

The purpose of paper is to present the problems connected with mathematical modelling the process of polymerization taking into consideration the real loads of drive.

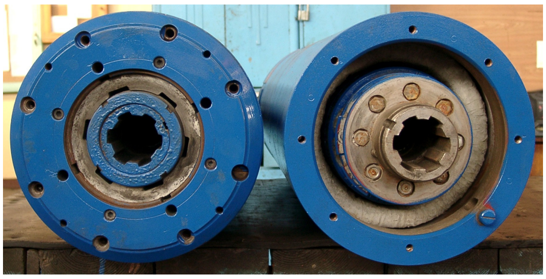

Fig. 1. The specially designed induction motors SAR-55/1500/09 after tests in polymerization reactor

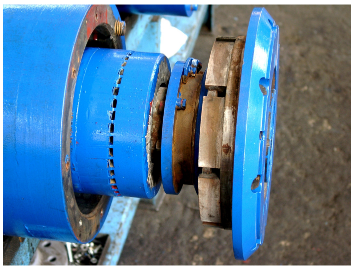

Fig. 2. A lower bearing of prototype SAR-55/1500/09/P2

2. THE POWER UNIT OF POLYMERIZATION REACTOR

The power unit including driving motor together with mixer works inside two reactor chambers. The specially designed induction motor together with multi-slot bipartite clutch is fixed in upper reactor chamber. The shaft of mixer together with separated connector and three-arm mixer is placed in lower reactor chamber. Simplified structure of power unit of polymerization reactor is depicted in Fig. 3 [2 – 6].

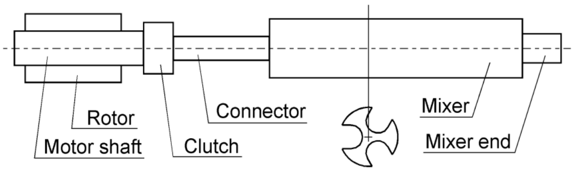

Fig. 3. Simplified structure of power unit of polymerization reactor

3. MODELLING THE FRICTION IN LOWER BEARING OF INDUCTION MOTOR

A loss of energy connected with conversion of certain form of energy into other forms occurs in each dynamic system. A precise quantitative depiction of friction forces is difficult because of dependence on velocity, shift, time and deformations of system elements. Thus, the dependences given below only approximate real character of those forces.

A dry-friction force in lower bearing of motor on which is suspended the whole rotating mass in vertical system creates anti-torque. This force is not applied lineally in the same r distance from rotor axis but this is distributed at various distances beginning from internal circumference of both contacting fricative surfaces belonging to lower slide bearing up to external circumference [1]. The various distances of applying the elementary friction forces should be considered to determine the whole moment of friction in the bearing. The assumption that the friction force is independent of surface area is usually taken into consideration. The friction force depends on both the force of gravity N perpendicular to the surface and friction factor μ. Thus, the moment of dry friction in lower bearing may be expressed as follows:

$$T_f = \begin{cases} -r\mu N & \text{for} \quad v > 0 \\ r\mu N & \text{for} \quad v < 0 \end{cases} \tag{1}$$

In general case the factor μ is a function of linear velocity v which may be associated with angular velocity ($v = r\omega$). Assuming that distribution of force N is regular on the whole fricative surface, the force density (force by surface) may be defined as:

$$n_A = \frac{N}{A} \tag{2}$$

where surface $A = \frac{\pi}{4}\left(d_e^2 - d_i^2\right) = \pi\left(r_e^2 - r_i^2\right)$,

whereas r_e, r_i are radiuses of external and internal circumference of fricative surfaces.

Elementary frictional force dF at distance r away from axis of revolution, for $v > 0$:

$$dF = -\mu n_A dA \tag{3}$$

where $dA = 2\pi r dr$ is elementary surface, whereas elementary moment of friction is determined as $dTf = rdF$. Therefore, for positive velocity:

$$T_f = -2\pi n_A \int_{r_i}^{r_e} \mu r^2 dr \tag{4}$$

An influence of parameter r on friction factor μ may be omitted if values of radiuses r_e and r_i are similar. Thus, only influence of angular velocity on friction factor μ may be considered:

$$T_f \approx -2\pi n_A \mu \int_{r_i}^{r_e} r^2 dr = -\frac{2}{3}\mu N \frac{r_e^3 - r_i^3}{r_e^2 - r_i^2} \tag{5}$$

Simplifying the problem it may be written that:

$$T_f \approx -\mu N r_{av} = -\mu N \frac{r_e + r_i}{2} \tag{6}$$

The following approximation of bearing frictional characteristic is assumed:

$$\mu = \frac{\mu_s - \mu_r}{1 + av} + \mu_r = \frac{\mu_s - \mu_r}{1 + ar\omega} + \mu_r \qquad (7)$$

where μ_s, μ_r are static and kinetic dry-friction factors, a is constant. The dependence (7) is shown in Fig. 4.

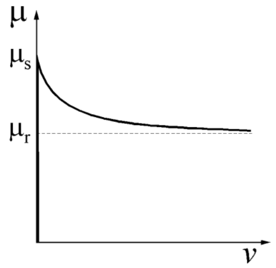

Fig. 4. Frictional characteristic of lower bearing according to assumed approximation

Replacing the friction factor in (6) with dependence (7) it may be derived that:

$$T_f \approx -N\left(r_e + r_i\right)\left[\frac{\mu_s - \mu_r}{2 + a\left(r_e + r_i\right)\omega} + \frac{\mu_r}{2}\right] \qquad (8)$$

4. MODELLING THE TORQUE RESULTING FROM POLYMERIZATION PROCESS

Ethylene $CH_2{=}CH_2$ is the simplest non-saturated hydrocarbon. A thermoplastic material – polyethylene is obtained as a result of ethylene polymerization. The polymerization process consists in synthesis of many molecules belonging to the same compound into single huge molecule with multiple molecular mass and different physical and chemical properties. The polymerization of ethylene takes place in gaseous phase under high pressure. Ethylene is colourless gas with molar mass comparable to dry air. In accordance with Avogadro's hypothesis, identical volumes of various gases (vapours) with the same temperature and pressure have the same number of molecules. It means that mass density of ethylene is comparable to mass density of dry air under the same conditions. Therefore, further studies concern dry air considered as substance in polymerization reactor. Collisions of mixer arms with polyethylene molecules are not taken into consideration.

The mass density of dry air is determined according to the following formula:

$$\rho_x = \rho_N \frac{T_N}{p_N} \cdot \frac{p_x}{T_x} \qquad (9)$$

where ρ_N = 1,293 kg/m^3, T_N = 273 K, p_N = 101325 Pa are conventional thermal parameters (natural physical conditions). There is a gas under pressure 2800×10^5 Pa and temperature 70 – 120 °C in reactor chamber. Mass density ρ_x of gas in reactor chamber at temperature of 70 °C is equal almost to 2900 kg/m^3.

A parallel between mixer in reactor chamber and centrifugal ventilator underlies the further considerations. Classifying ventilators as centrifugal, axial-flow and diagonal is connected with direction of substance flow through the rotor [10]. The direction of flow along the rotor axis is continued in axial-flow ventilators. In centrifugal ventilators the direction of flow is changed from axial to perpendicular to the rotor axis. The diagonal ventilators are classified as semi-axial and semi-centrifugal, and they possess features of both basic solutions. Geometrical parameters of mixer cross-section related to cross-section of centrifugal ventilator are depicted in Fig. 5.

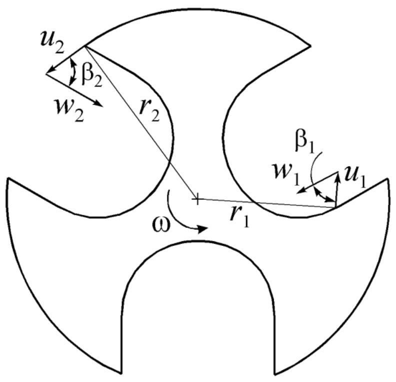

Fig. 5. Geometrical parameters of mixer, where: u_1, u_2 are circumferential velocities, w_1, w_2 are tangential components of velocities, ω is angular velocity of a mixer, r_1, r_2 are radiuses of circumferences

The tangential components w_1 and w_2 of velocities are directed inward the mixer. It depends on inclination of mixer arms. Thus, the substance is sucked in mixer slots perpendicularly to axis of revolution and gets out along the axis of revolution.

The frictional loss in rotor of ventilator is as follows [10]:

$$\Delta p_f \le \xi \frac{w_2^2}{2} \rho_x \qquad (10)$$

The values of friction factor ξ are chosen from the range of 0,2 to 0,25. The maximal delivery of ventilator for pressure increment (in reactor chamber) equal to zero is defined as [10]:

$$\dot{V} = 2\pi r_2 u_2 b_2 tg\beta_2 = 2\pi r_2^2 b_2 tg\beta_2 \omega \qquad (11)$$

where $u_2 = r_2\omega$, $\beta_2 = arccos\left(r_d / r_2\right)$, b_2 is mixer length. A method of angle β_2 calculation is depicted in Fig. 6.

The tangential component w_2 of velocity is given as follows:

$$w_2 = \left(0 \div 0{,}33\right) \cdot \frac{\tau_2 u_2}{cos\beta_2} = \left(0 \div 0{,}33\right) \cdot \frac{\tau_2 r_2^2 \omega}{r_d} \qquad (12)$$

where $\tau_2 = \pi \sin\beta_2 \left[\pi \sin\beta_2 - 3\sin\left(\beta_2 - \pi/6\right)\right]^{-1}$ is coefficient of narrowing a cross-section of three-arm mixer.

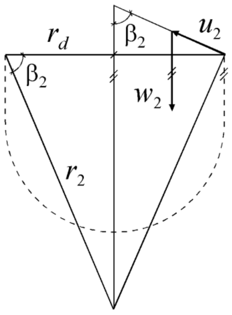

Fig. 6. Calculation of angle β_2

The drive of mixer has to cause the swell Δp compensating for the loss of pressure in reactor chamber as a result of friction of gas molecules with rotor arms. The output power of drive may be calculated as [10]:

$$P = \left(0 \div 0{,}33\right)\frac{\dot{V}\Delta p_f}{\eta} \tag{13}$$

The efficiency $\eta \approx 1$ may be assumed for closed reactor chamber. The load torque of drive

$$T_m = \frac{P}{\omega} \leq \left(0 \div 0{,}33\right)^3 \cdot \frac{\xi\pi\rho_x r_2^6 b_2 \tau_2^2 tg\beta_2}{\eta r_d^2} \cdot \omega^2 \tag{14}$$

From above dependence it results that load torque of drive is square function of angular velocity as the geometrical parameters of mixer are constant [1].

5. MATHEMATICAL MODEL OF POWER UNIT

The vector equations are commonly used in mathematical description of induction machines [1, 12]. The vector equations of induction machine expressed in Cartesian coordinate system rotating with any angular velocity ω_a are as follows:

$$\underline{u}_s = R_s \underline{i}_s + \frac{d}{dt}\underline{\psi}_s + j\omega_a \underline{\psi}_s \tag{15}$$

$$\underline{u}_r = R_r \underline{i}_r + \frac{d}{dt}\underline{\psi}_r + j(\omega_a - p\omega_m)\underline{\psi}_r \tag{16}$$

where $\underline{u}_s$, $\underline{u}_r$ are vectors of stator and rotor voltages, $\underline{i}_s$, $\underline{i}_r$ are vectors of stator and rotor currents, $\underline{\psi}_s$, $\underline{\psi}_r$ are vectors of stator and rotor fluxes, R_s, R_r are resistances of stator and rotor windings, ω_m is rotor angular velocity, p is number of magnetic couple pairs.

The dependence $\omega_a = 0$ should be taken into consideration if the equations are expressed in immovable coordinate system. Considering a squirrel-cage motor also the dependence $\underline{u}_r = 0$ should be taken into consideration, thus:

$$\frac{d}{dt}\underline{\psi}_s = -R_s \underline{i}_s + \underline{u}_s \tag{17}$$

$$\frac{d}{dt}\underline{\psi}_r = -R_r \underline{i}_r + jp\omega_m \underline{\psi}_r \tag{18}$$

The equations (17) and (18) should be completed with the following dependences:

$$\underline{\psi}_s = L_s \underline{i}_s + L_m \underline{i}_r \tag{19}$$

$$\overline{\psi}_r = L_r \underline{i}_r + L_m \underline{i}_s \tag{20}$$

and equation of rotating masses motion:

$$\theta_m \frac{d\omega_m}{dt} = t_e - t_f - t_m \tag{21}$$

$$t_e = p\, Im\left(\underline{\psi}_s^* \cdot \underline{i}_s\right) \tag{22}$$

where t_e is electromagnetic torque on shaft of induction motor , t_f is moment of friction and t_m is load torque resulting from polymerization process, L_s, L_r are self-inductances of stator and rotor, L_m is magnetization inductance, θ_m is moment of rotor inertia.

In kinematic structure of considered drive the mixer in polymerization reactor is discreetly segmented [4 - 6, 11] whereas the connector between rotor and mixer is described with application of concentrated parameters. The mixer is segmented into five equal sections. The length of each section is comparable to length of rotor axis and connector. The same mass and the same elastic constant are attributed to each section of segmented mixer. The results of digital simulation made for kinematic structure containing elements segmented into several sections are similar to results made for kinematic structure containing elements segmented into infinity number of sections [11]. It corresponds to the wave model. The equations of dynamic equilibrium of power unit of polymerization reactor are expressed as (23) [4 – 6]. Discrete load torques are expressed as $t_i = c\omega_i^2$, $i = 2,\ldots,6$, where c is constant of proportionality resulting from consideration in chapter 4,

$$\theta_m \frac{d\omega_m}{dt} + k_1\left(\varphi_m - \varphi_1\right) = t_e - t_f\left(\omega_m\right)$$

$$\theta_1 \frac{d\omega_1}{dt} + k_1\left(\varphi_1 - \varphi_m\right) + k_2\left(\varphi_1 - \varphi_2\right) = 0$$

$$\theta_2 \frac{d\omega_i}{dt} + k_2\left(\varphi_i - \varphi_{i-1}\right) + k_2\left(\varphi_i - \varphi_{i+1}\right) = -t_i\,,\ i = 2,\ldots,5$$

$$\theta_2 \frac{d\omega_6}{dt} + k_2(\varphi_6 - \varphi_5) = -t_6$$

$$\frac{d\varphi_m}{dt} = \omega_m, \quad \frac{d\varphi_i}{dt} = \omega_i, \; i = 1,\ldots,6 \qquad (23)$$

while θ_m, θ_l, $5\theta_2$ are moments of rotor, connector and mixer inertia, ω_m, ω_i, φ_m, φ_i are angular velocities and angles of torsion, k_1, k_2 are elastic factors.

6. EXAMPLES OF TRANSIENT RESPONSES AND TRAJECTORIES

Digital simulations of driving unit for polymerization reactor have been made on the basis of presented mathematical model considering real loads. The examples of transient responses of specially designed induction motor connected directly to the power grid are shown in Fig. 7 and 10. The trajectories made for selected operating conditions of driving unit are shown in Fig. 8, 9, 11 and 12.

Discrete load torques are distributed along the mixer axis. Results of simulations of driving unit when discrete load torques depend on angular velocities are shown in Fig. 7 – 9 while results of simulations at constant discrete load torques are shown in Fig. 10 – 12.

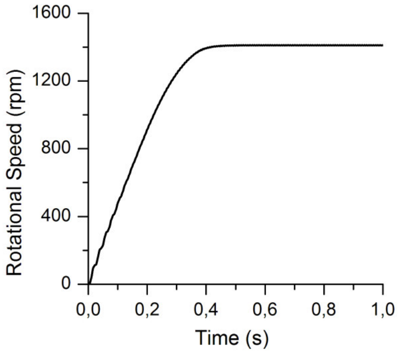

Fig. 7. Time-variability of motor rotational speed

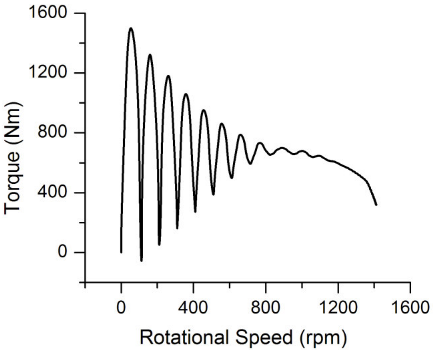

Fig. 8. Trajectory of electromagnetic torque on motor shaft as a function of rotational speed

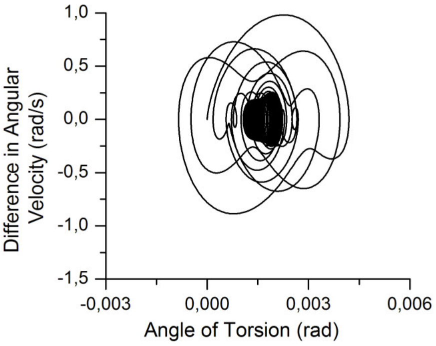

Fig. 9. Trajectory of difference in angular velocity at upper and lower end of connector as a function of angle of torsion

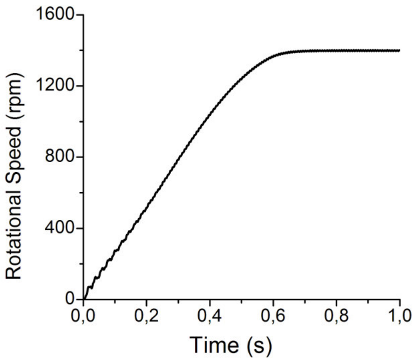

Fig. 10. Time-variability of motor rotational speed

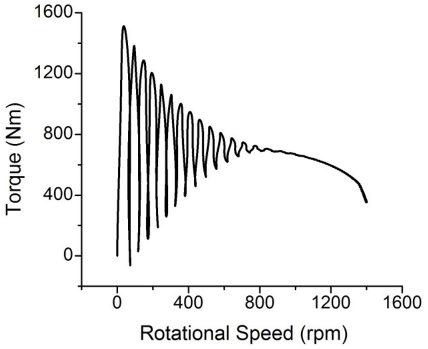

Fig. 11. Trajectory of electromagnetic torque on motor shaft as a function of rotational speed

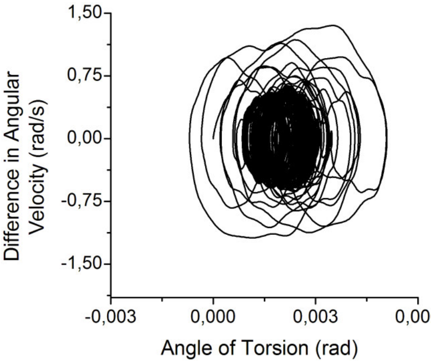

Fig. 12. Trajectory of difference in angular velocity at upper and lower end of connector as a function of angle of torsion

7. CONCLUSIONS

Developing the mathematical models of power transmission systems considering real loads of electric drives is significant in order to determine the directives required in designing the systems containing specially designed induction motors and working under real operating conditions. Moreover, mathematical modelling and digital simulations are useful in optimization of electric drives working under real operating conditions.

The numerical analysis was made on the basis of formulated mathematical model in order to develop and construct the prototypical specially designed induction motors [9]. The results of simulations may be applied in practice in order to determine the maximal loads occurring in connectors of rotating elements in reactor chamber. The ranges of angles of torsion and the highest values of torques on motor shaft and time of stabilization of angular velocities of rotating elements in reactor chamber are significant in determining the principles and instructions in order to design electric drives for polymerization reactors [9].

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